I am a Legend
Celine & Elie Bursztein
https://www.elie.net/hs

Digital Collectible Card Game

Released by Blizzard in 2014

Based of World of Warcraft universe

Painting by http://www.jason-w.com/blog/the-fall-of-lordaeron

Sometimes too interesting leads to un-intended consequences

Game complexity generates exploitable biases

Outline

Outline

1. Finding undervalued cards

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2. Predicting opponent deck

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2. Predicting opponent deck
3. ~~Predicting the game outcome~~

Outline

1. Finding undervalued cards
2. Predicting opponent deck
3. ~~Predicting the game outcome~~
4. ~~Incoming alien invasion (or not)~~

Heigan the Unclean
0/3
45
END TURN
LightWind
1/4
16:28
Azure Drake
Conceal
Leeroy Jenkins
Charge. Battlecry:
Earthen Ring Farseer

You hero

Heigan the Unclean
0/3
45
END TURN
LightWind
1/4
16:28
Azure Drake
Conceal
Leeroy Jenkins
Charge. Battlecry:
Earthen Ring Farseer

Opponent hero

You hero

Hero health

Heigan the Unclean
0/3
45
END TURN
LightWind
1/4
16:28
Deck

Heigan the Unclean
0/3
45
END TURN
LightWind
1
16:28
1/4
Azure Drake
Conceal
Leeroy Jenkins
Charge. Battlecry:
Earthen Ring Farseer

hand

hand

Heigan the Unclean
0/3
45
3
2
3
4
3
4
END TURN
6
6
1
2
1
18
2
LightWind
5
1
4
3
1/4
1
16:28

Heigan the Unclean
0/3
45
END TURN
LightWind
1/4
16:28
Conceal
Leeroy Jenkins
Charge. Battlecry:
Earthen Ring Farseer

mana pool

Weapon

Heigan the Unclean
0/3
45
END TURN
LightWind
1/4
16:28
Azure Drake
Conceal
Leeroy Jenkins
Charge. Battlecry:
Earthen Ring Farseer

Minions

Heigan the Unclean
0/3
45
END TURN
LightWind
1/4
16:28
Azure Drake
Conceal
Leeroy Jenkins
Charge. Battlecry:
Earthen Ring Farseer

0/3
30
2
Reinforce
Hero Power
Summon a 1/1 Silver Hand Recruit.
ENEMY TURN
30
2
0/2
Wrath
Molten Giant
Healing Touch
Ironbark Protect
Claw

0/3
30
2
Reinforce
Hero Power
Summon a 1/1 Silver Hand Recruit.
ENEMY TURN
30
2
0/2
Wrath
Molten Giant
Healing Touch
Ironbark Protect
Claw

4
Chillwind Yeti
4
5

4
Chillwind Yeti
4
5

Mana

Mana

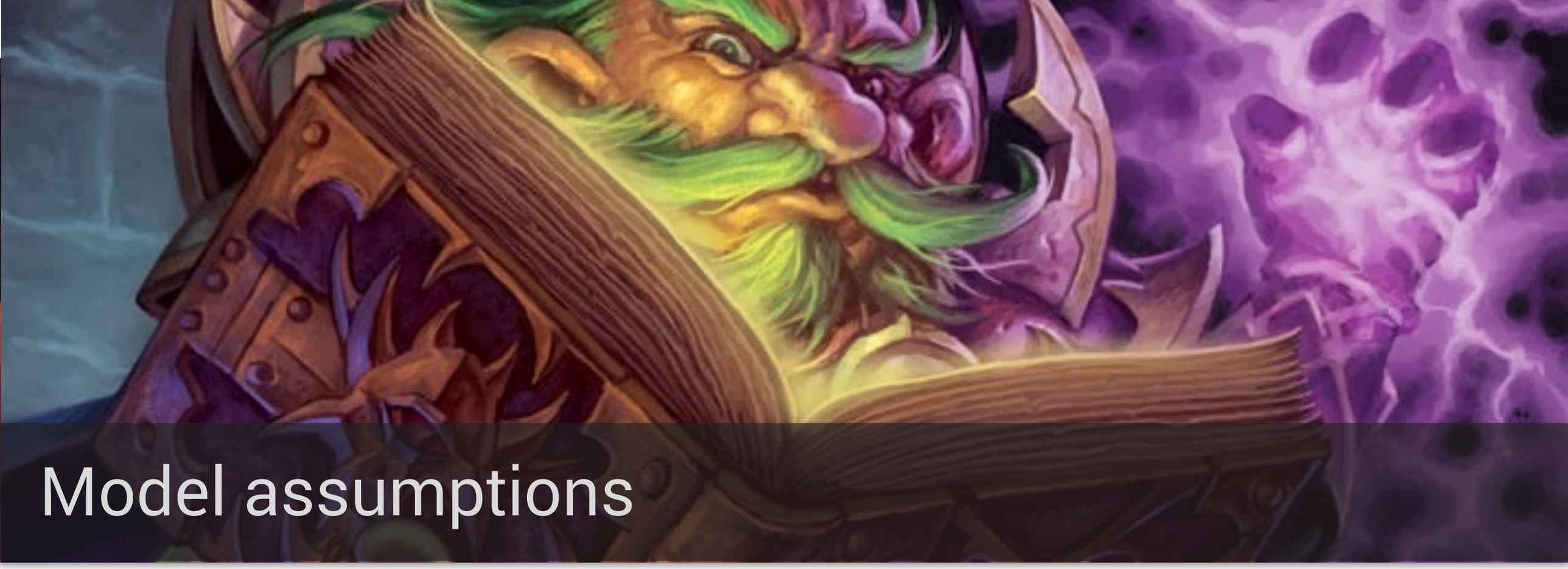
Attack

Mana
4
Chillwind Yeti
Attack
4
5
Health

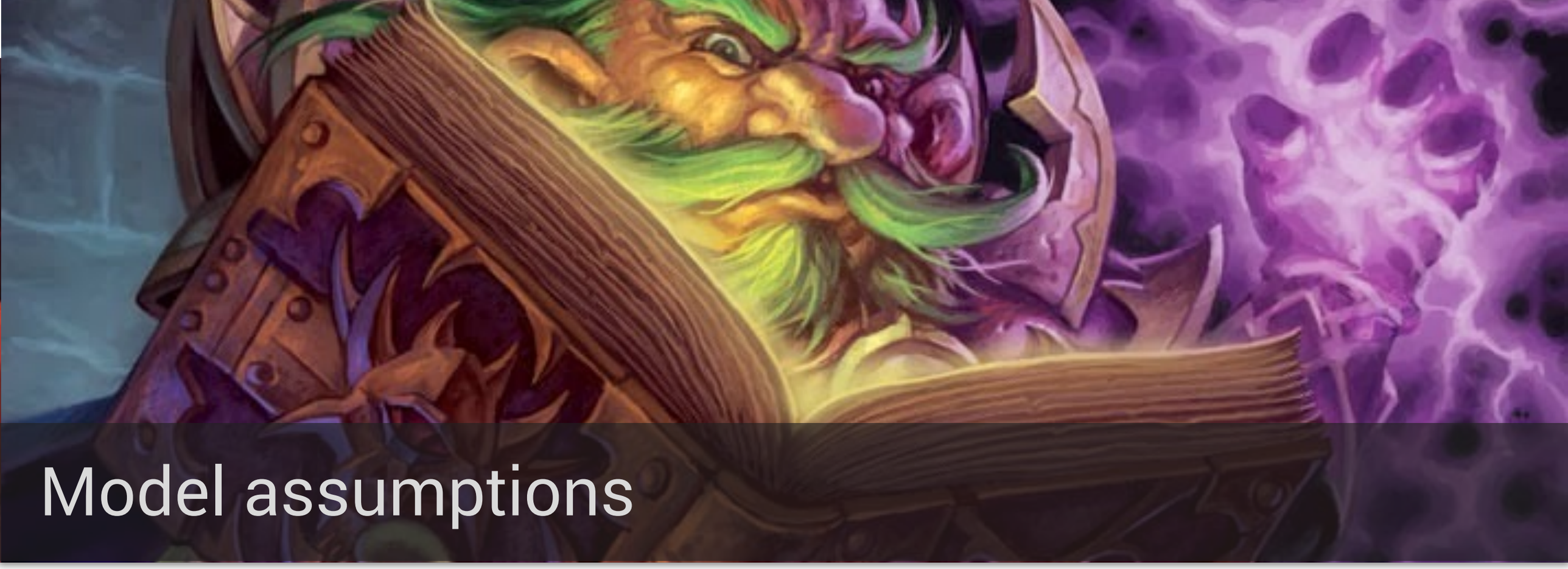
Card special abilities is what makes the game complex and interesting

Card special abilities is what makes the game complex and interesting

Finding undervalued cards

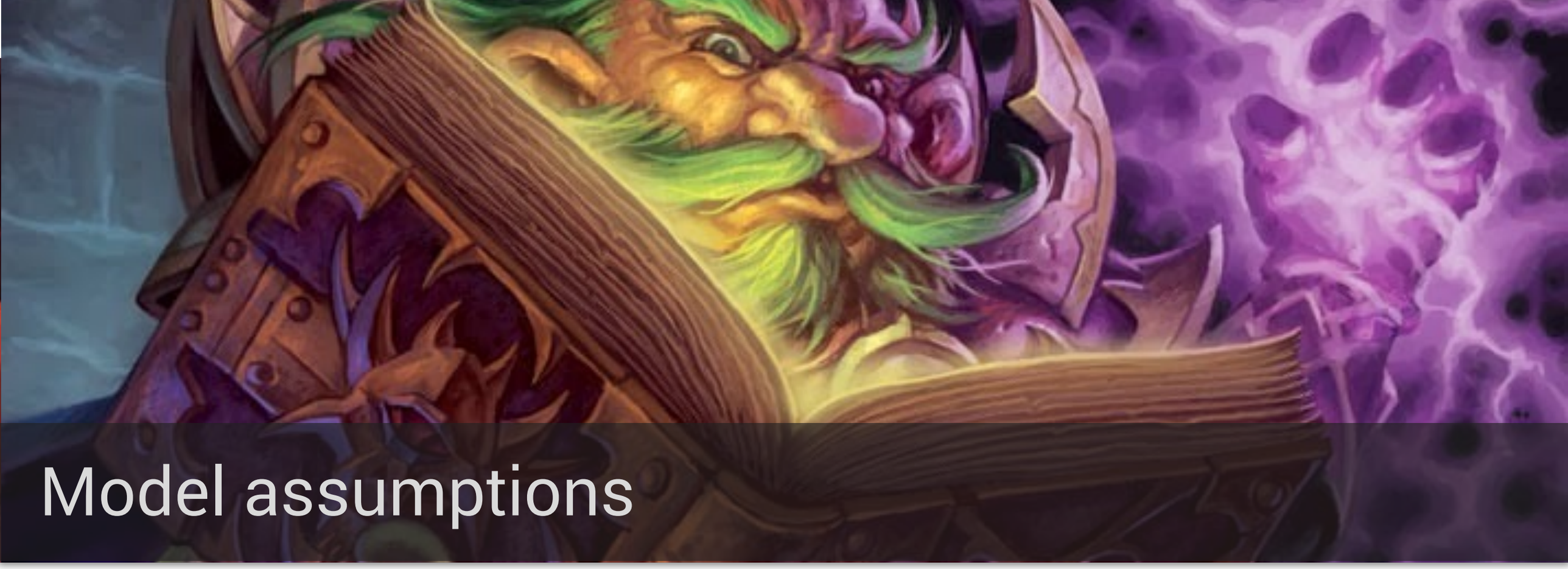


Model assumptions



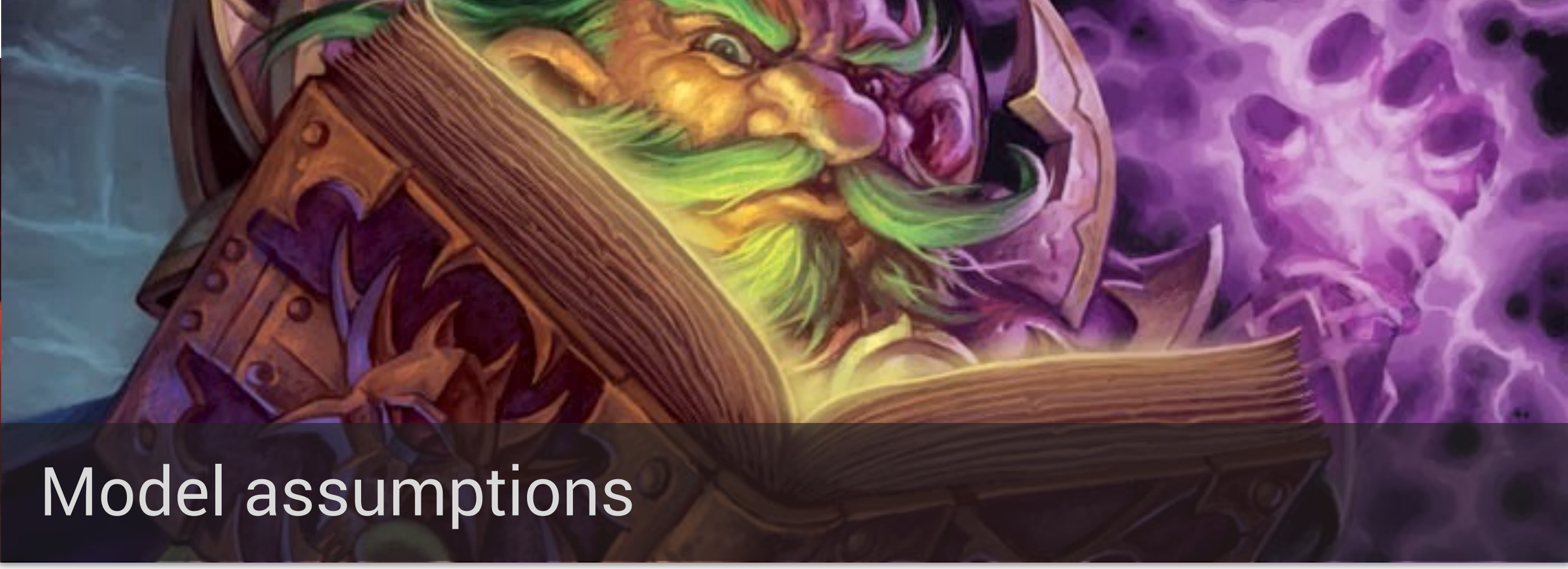
Model assumptions

1. Mana cost is proportional to card power



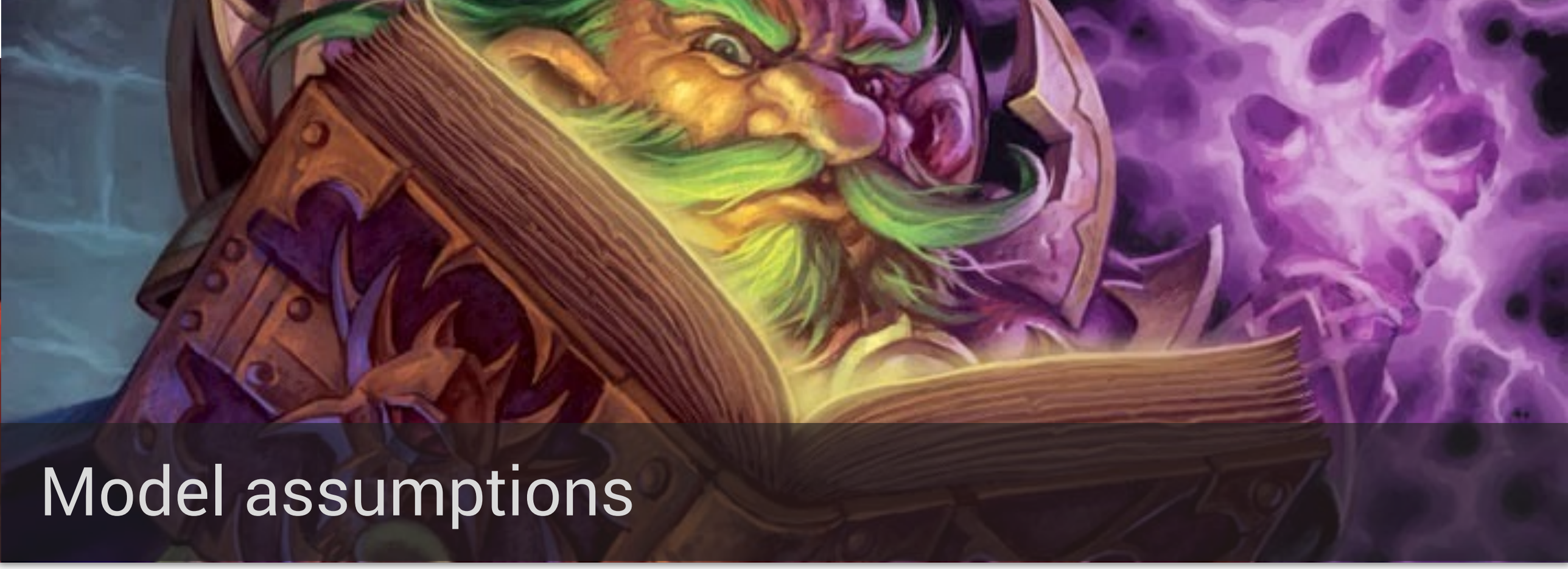
Model assumptions

1. Mana cost is proportional to card power
2. The power of cards roughly increase linearly



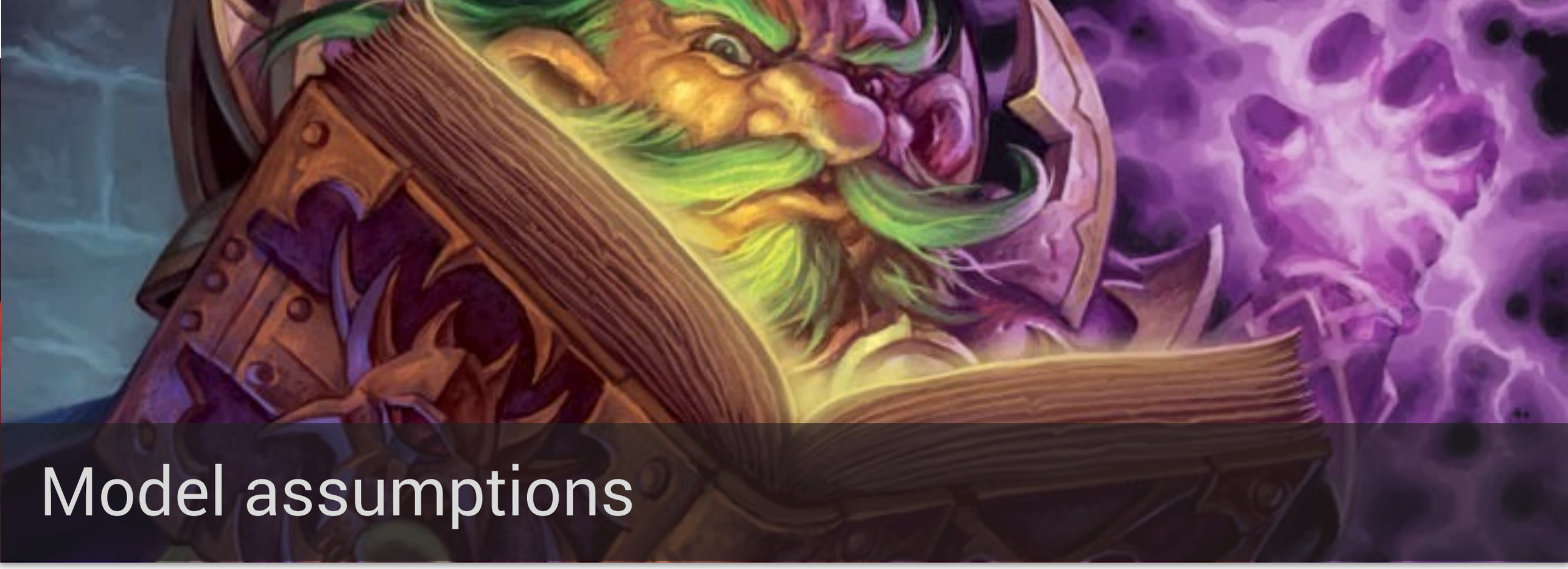
Model assumptions

1. Mana cost is proportional to card power
2. The power of cards roughly increase linearly
3. Card effects have constant price



Model assumptions

1. Mana cost is proportional to card power
2. The power of cards roughly increase linearly
3. Card effects have constant price
4. A card have an intrinsic value



Model assumptions

1. Mana cost is proportional to card power
2. The power of cards roughly increase linearly
3. Card effects have constant price
4. A card have an intrinsic value
5. The value of the card is the sum of its attribute

mana = attack + health + intrinsic value

$$\text{mana} = \text{attack} + \text{health} + \text{intrinsic value}$$

$$4 = 4a + 5h + i$$

6
Boulderfist Ogre
6
7

$$6 = 6a + 7h + i$$

$$6 = 6a + 7h + i$$

$$6 = 6a + 7h + i$$

$$4 = 4a + 5h + i$$

$$6 = 6a + 7h + i$$

$$\downarrow /6$$

$$4 = 4a + 5h + i$$

$$6 = 6a + 7h + i$$

$$\downarrow /6$$

$$1 = 1a + 1.16h + i$$

$$4 = 4a + 5h + i$$

$$6 = 6a + 7h + i$$

$$\downarrow /6$$

$$1 = 1a + 1.16h + i$$

$$4 = 4a + 5h + i$$

$$\downarrow /4$$

$$6 = 6a + 7h + i$$

$$\downarrow /6$$

$$1 = 1a + 1.16h + i$$

$$4 = 4a + 5h + i$$

$$\downarrow /4$$

$$1 = 1a + 1.25h + i$$

4
Fireball
Deal 6 damage.

4 = 6d

4 = 6d ⟶ 1 mana = 1.5 dmg

4 = 6d ⟶ 1 mana = 1.5 dmg

4 = 6d ⟶ 1 mana = 1.5 dmg

10 = 10d

4 = 6d ⟶ 1 mana = 1.5 dmg

10 = 10d ⟶ 1 mana = 1 dmg

4 = 6d ⟶ 1 mana = 1.5 dmg

10 = 10d ⟶ 1 mana = 1 dmg

Pre nerf (8 mana)

4 = 6d ⟶ 1 mana = 1.5 dmg

10 = 10d ⟶ 1 mana = 1 dmg

Pre nerf (8 mana)

8 = 10d

4 = 6d → 1 mana = 1.5 dmg

10 = 10d → 1 mana = 1 dmg

Pre nerf (8 mana)

8 = 10d → 1 mana = 1.25 dmg

10
Pyroblast
Deal 10 damage.

4
Fireball
Deal 6 damage.

10 damages imply 4 damages

10 damages imply 4 damages

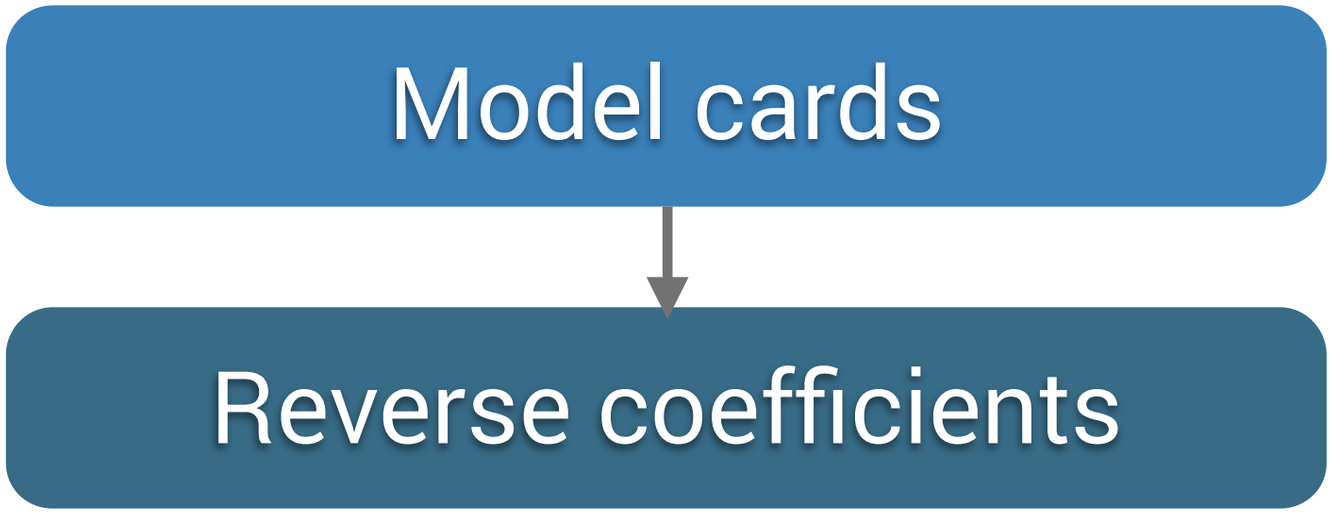
15 damages imply 6 damages

Hunting for under-valued cards

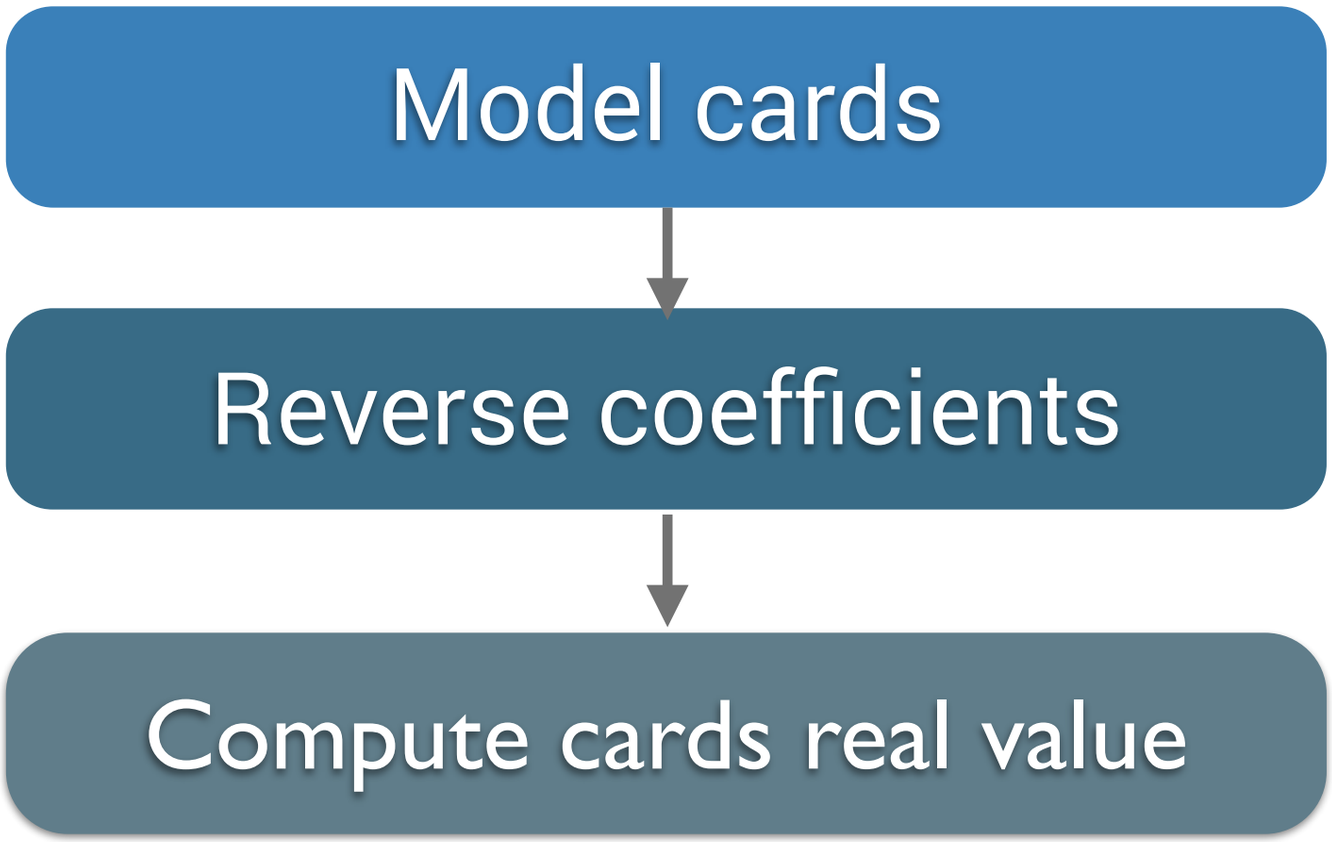
How to find undervalued cards?

Model cards

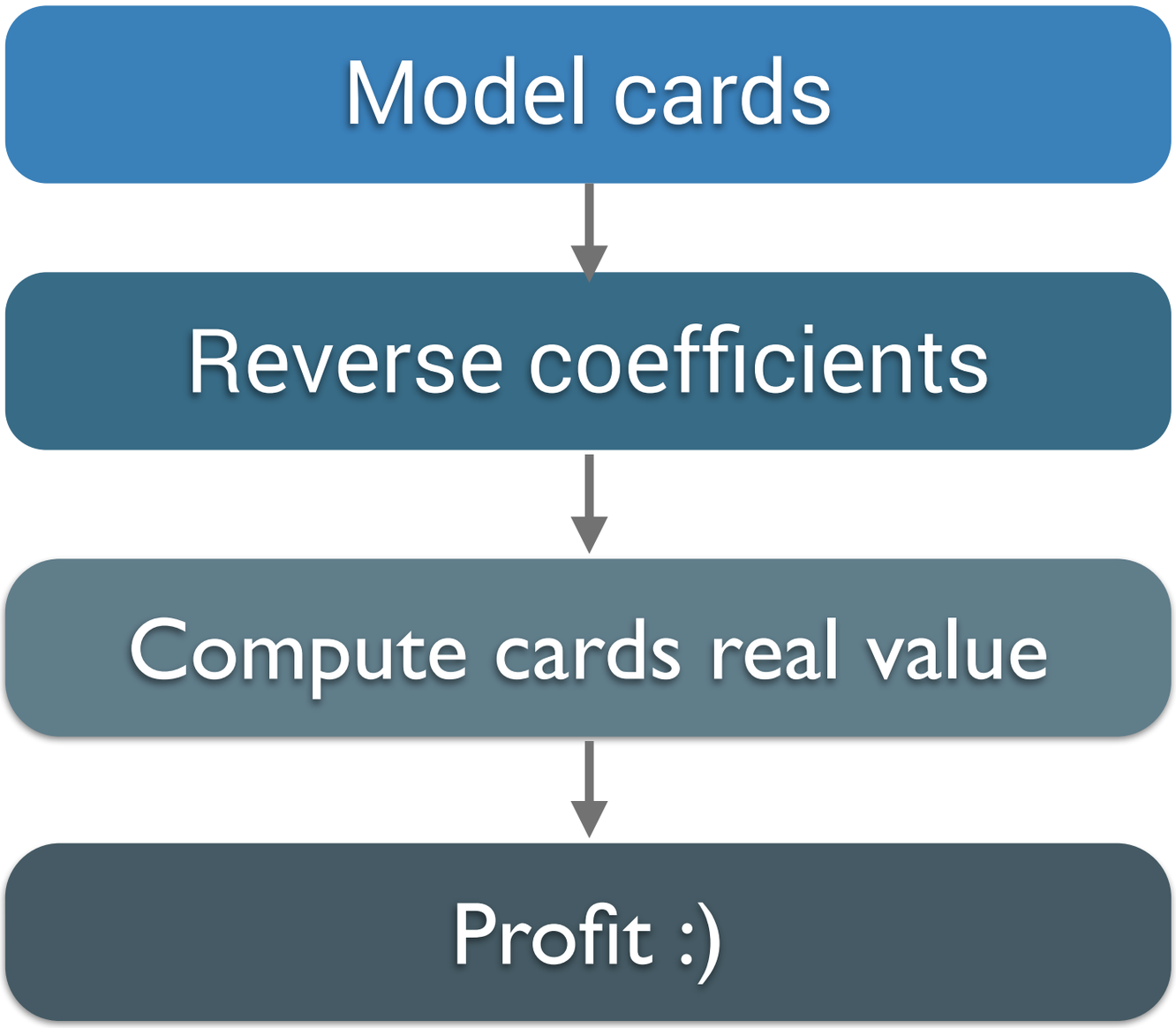
How to find undervalued cards?



How to find undervalued cards?



How to find undervalued cards?



Approach illustrated

4
Kor'kron Elite
Charge
4
3

Charge

6
Argent Commander
Charge, Divine Shield
4
2

6
Reckless Rocketeer
Charge
5
2

Charge

Divine shield

4
Kor'kron Elite
Charge
4
3

6
Argent Commander
Charge, Divine Shield
4
2

3
Scarlet Crusader
Divine Shield
3
1

6
Reckless Rocketeer
Charge
5
2

1
Argent Squire
Divine Shield
1
1

4
Kor'kron Elite
Charge
4
3

3
Scarlet Crusader
Divine Shield
3
1

$$4 = 4a + 3h + c + i$$

6
Argent Commander
Charge, Divine Shield
4
2

6
Reckless Rocketeer
Charge
5
2

1
Argent Squire
Divine Shield
1
1

4 = 4a + 3h + c + i

3 = 3a + 1h + d + i

6 = 5a + 2h + c + i

6 = 4a + 2h + c + d + i

1 = 1a + 1h + d + i

Reversing attribute cost

Reversing attribute cost

Reversing attribute cost

mana Atk Health Charge Divine Intrinsic

Reversing attribute cost

mana	Atk	Health	Charge	Divine	Intrinsic
4	4	3	1	0	1

Reversing attribute cost

mana	Atk	Health	Charge	Divine	Intrinsic
4	4	3	1	0	1
6	5	2	1	0	1

Reversing attribute cost

mana	Atk	Health	Charge	Divine	Intrinsic
4	4	3	1	0	1
6	5	2	1	0	1
6	4	2	1	1	1

Reversing attribute cost

mana	Atk	Health	Charge	Divine	Intrinsic
4	4	3	1	0	1
6	5	2	1	0	1
6	4	2	1	1	1
3	3	1	0	1	1

Reversing attribute cost

mana	Atk	Health	Charge	Divine	Intrinsic
4	4	3	1	0	1
6	5	2	1	0	1
6	4	2	1	1	1
3	3	1	0	1	1
1	1	1	0	1	1

Reversing attribute cost

mana	Atk	Health	Charge	Divine	Intrinsic
4	4	3	1	0	1
6	5	2	1	0	1
6	4	2	1	1	1
3	3	1	0	1	1
1	1	1	0	1	1

Least square

Reversing attribute cost

mana	Atk	Health	Charge	Divine	Intrinsic
4	4	3	1	0	1
6	5	2	1	0	1
6	4	2	1	1	1
3	3	1	0	1	1
1	1	1	0	1	1

Least square

Atk	=	1
Health	=	-1
Charge	=	2
Divine	=	1
Intrinsic	=	1

Reversing attribute cost

mana	Atk	Health	Charge	Divine	Intrinsic
4	4	3	1	0	1
6	5	2	1	0	1
6	4	2	1	1	1
3	3	1	0	1	1
1	1	1	0	1	1

Least square →

Atk	=	1
Health	=	-1
Charge	=	2
Divine	=	1
Intrinsic	=	1

Warning these example attribute costs are bogus as we didn’t use enough cards

Finding card real price using reversed coefficients

Coeffs: Charge = 2 Atk = 1 Health = -1 Divine = 1 Intrinsic = 1

Finding card real price using reversed coefficients

$$4a + 2h + c + d + i$$

Coeffs: Charge = 2 Atk = 1 Health = -1 Divine = 1 Intrinsic = 1

Finding card real price using reversed coefficients

$$4a + 2h + c + d + i$$

$$4*1 + 2*-1 + 2 + 1 + 1 = 6$$

Coeffs: Charge = 2 Atk = 1 Health = -1 Divine = 1 Intrinsic = 1

Finding card real price using reversed coefficients

4a + 2h + c + d + i

4*1 + 2*-1 + 2 + 1 + 1 = 6

Coeffs: Charge = 2 Atk = 1 Health = -1 Divine = 1 Intrinsic = 1

Finding card real price using reversed coefficients

4a + 2h + c + d + i

4*1 + 2*-1 + 2 + 1 + 1 = 6

1a + 1h + d + i

Coeffs: Charge = 2 Atk = 1 Health = -1 Divine = 1 Intrinsic = 1

Finding card real price using reversed coefficients

4a + 2h + c + d + i

4*1 + 2*-1 + 2 + 1 + 1 = 6

1a + 1h + d + i

1*1 + 1*-1 + 1 + 1 = 2

Coeffs: Charge = 2 Atk = 1 Health = -1 Divine = 1 Intrinsic = 1

Finding card real price using reversed coefficients

4a + 2h + c + d + i

4*1 + 2*-1 + 2 + 1 + 1 = 6

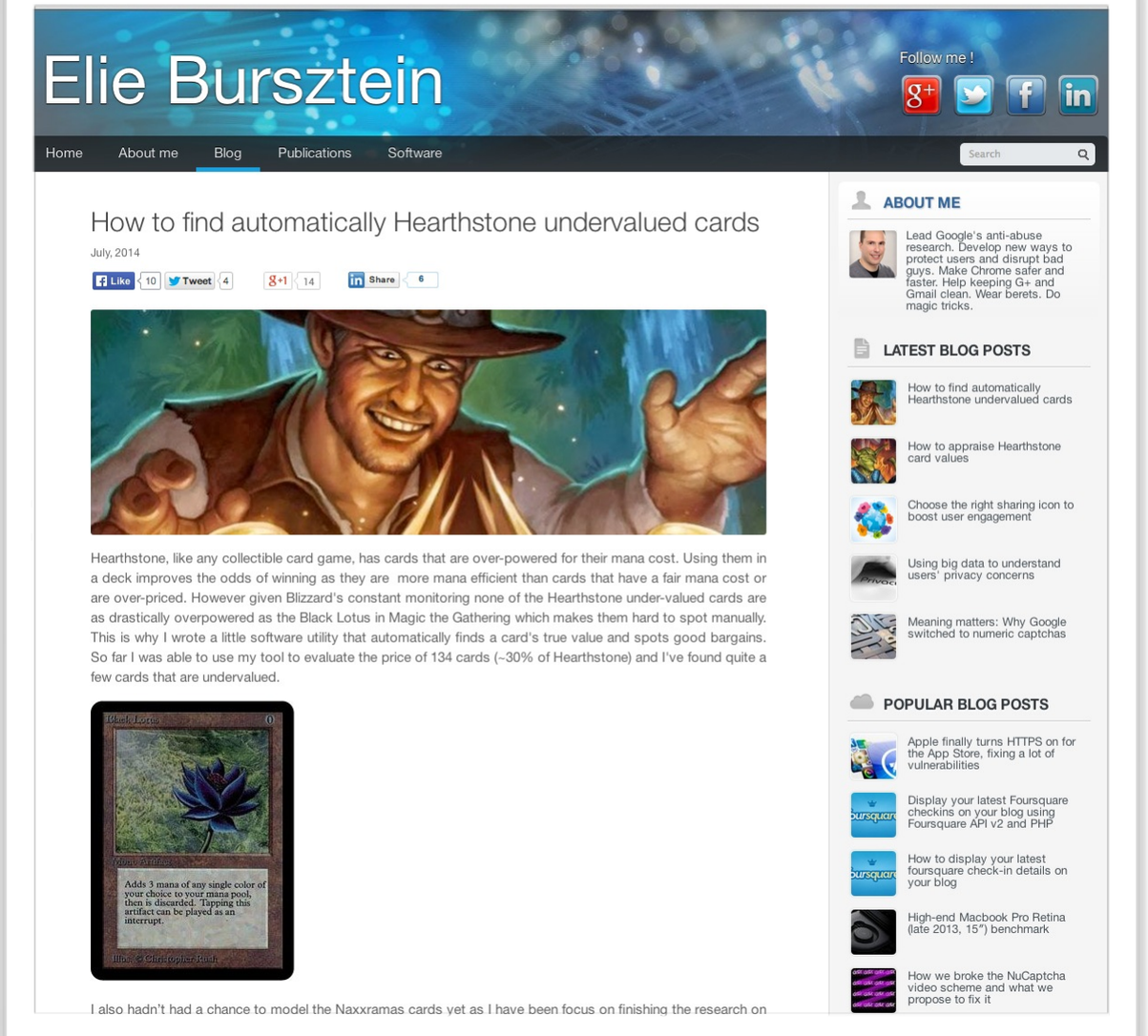
Under-valued card!

1a + 1h + d + i

1*1 + 1*-1 + 1 + 1 = 2

Coeffs: Charge = 2 Atk = 1 Health = -1 Divine = 1 Intrinsic = 1

Thanks you for the feedback!



https://www.elie.net/hs

Modeling dependance between characteristics

Thanks to Niels for the idea

Modeling dependance between characteristics

Charge = Atk * charge coeff

Windfury = Atk * windfury coeff

Divine = ? (health related?)

Thanks to Niels for the idea

Modeling dependance between characteristics

Charge = Atk * charge coeff

Windfury = Atk * windfury coeff

Divine = ? (health related?)

Model also use a card budget:
2*mana + 1

Thanks to Niels for the idea

Et voila!

Reversed budget coefficients

Effect	cost per point
Destroy minion	10.63
Board damage	3.69
Draw card	3.68
Divine Shield	2.74
Freeze	2.04
Silence	1.66
Damage	1.64
Durability	1.22
Stealth	1.21
Attack	1.14
Taunt	1.02
WindFury	0.96
SpellPower	0.93
Health	0.81
Battlecry heal	0.69
Battlecry self hero heal	0.68
Charge	0.65
Intrinsic value	0.32

Reversed budget coefficients

Effect	cost per point
Destroy minion	10.63
Board damage	3.69
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2 coeff point ~ 1 mana point

Reversed budget coefficients

Effect	cost per point
Destroy minion	10.63
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Charge	0.65
Intrinsic value	0.32

Effect	Cost per point
Opponent draw card	-3.97
Discard cards	-2.67
Overload	-1.68
Self hero damage	-0.54

2 coeff point ~ 1 mana point

Reversed budget coefficients

Effect	cost per point
Destroy minion	10.63
Board damage	3.69
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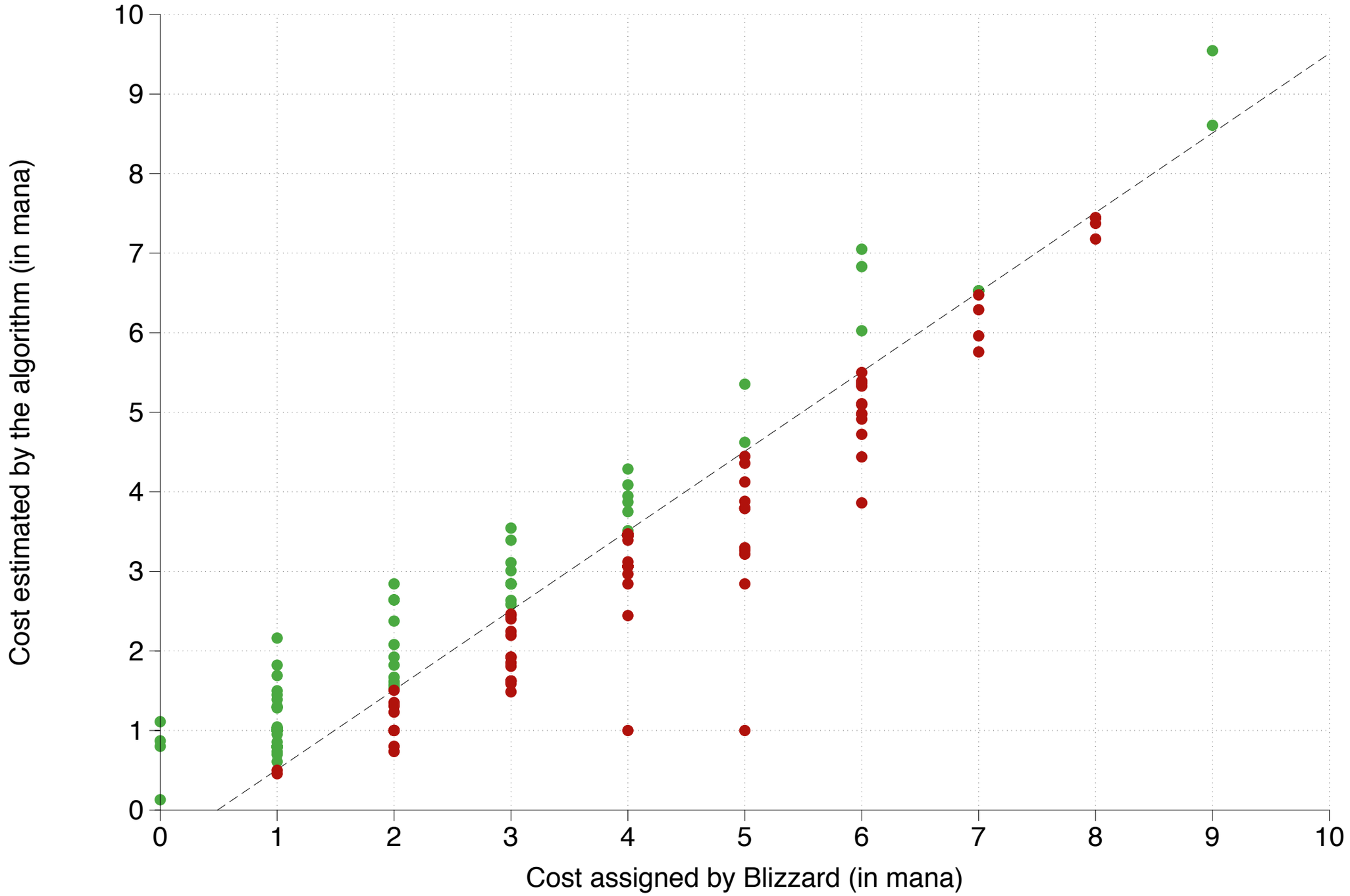
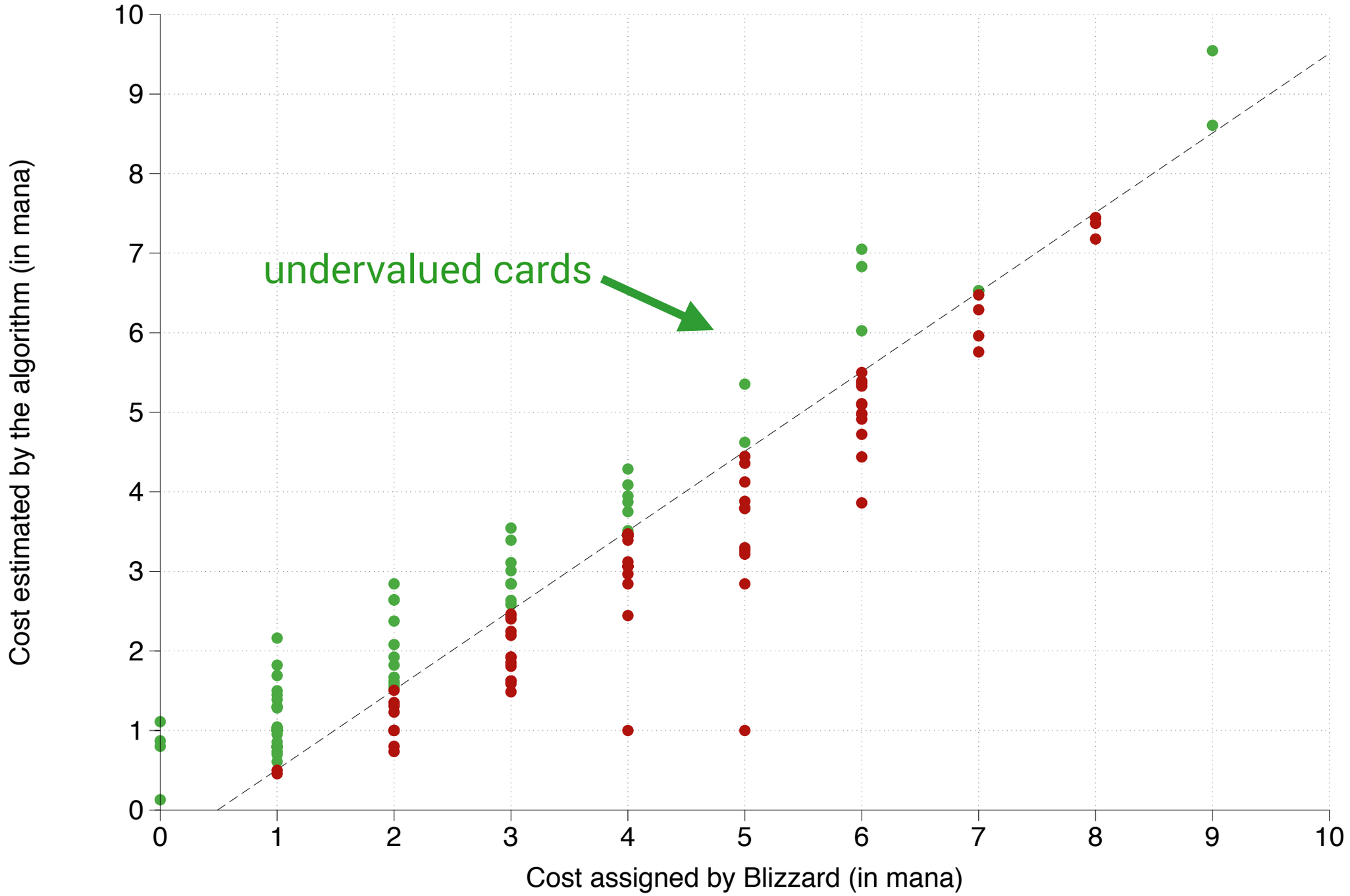
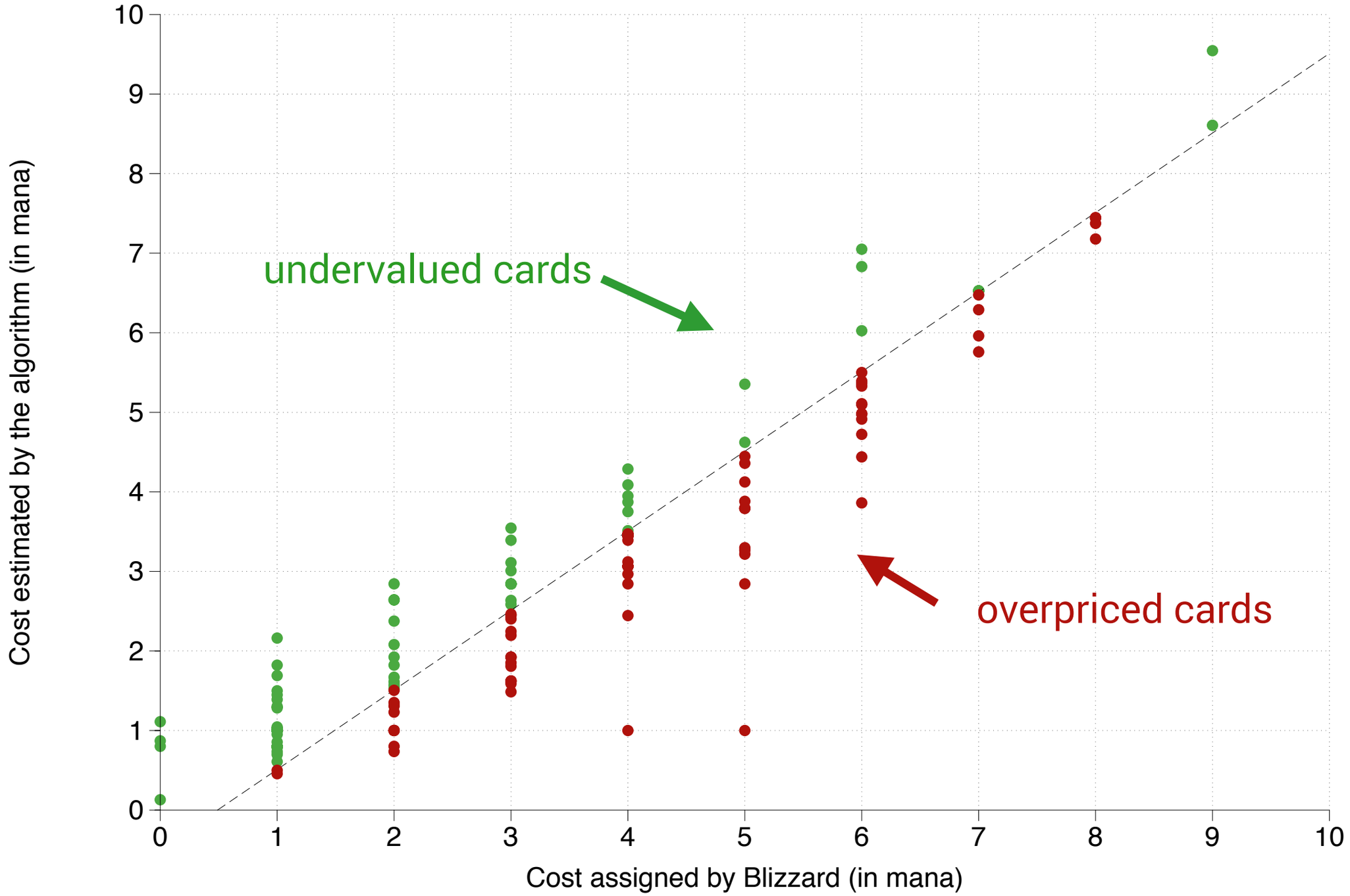
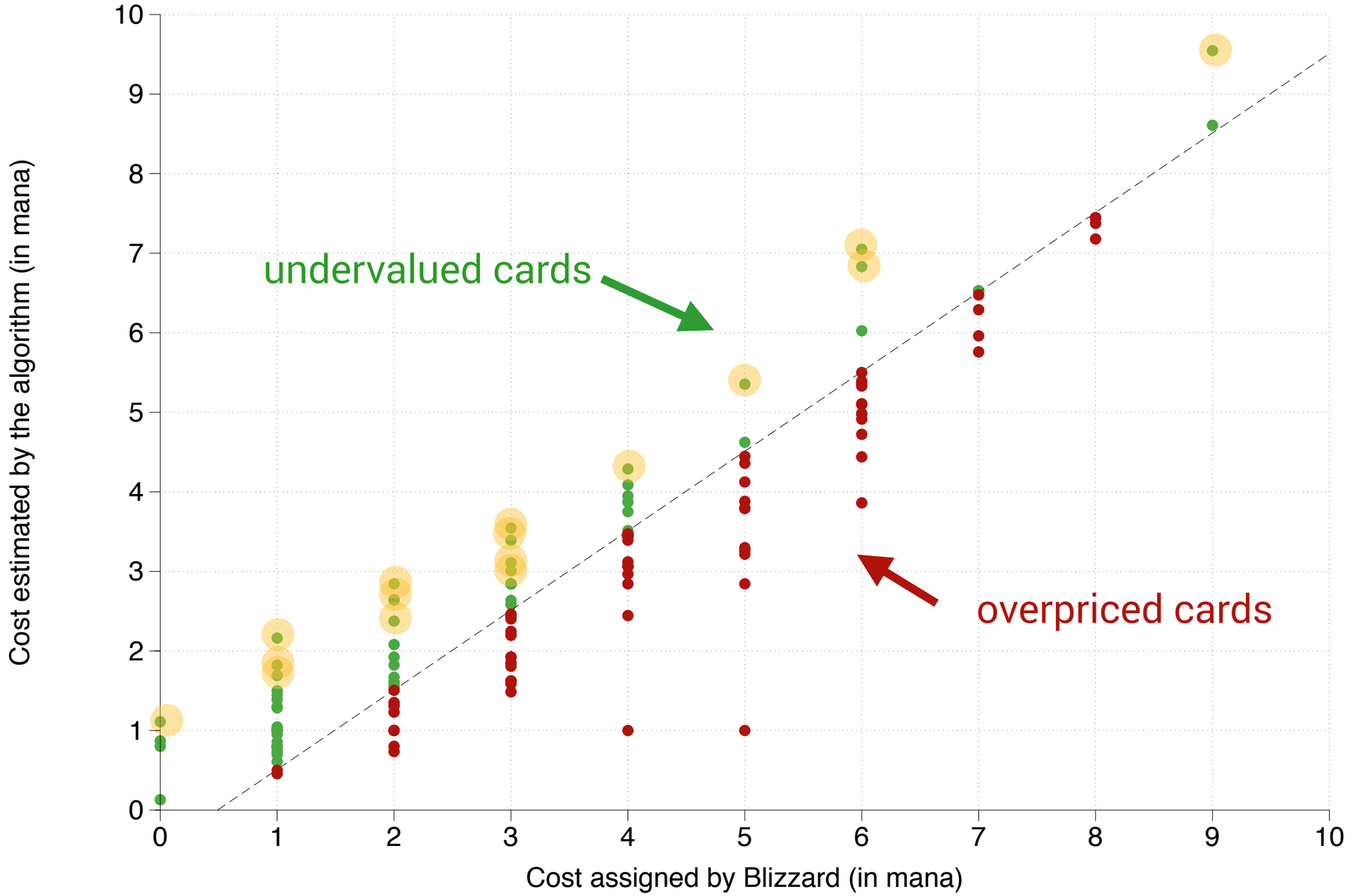

Cost estimated by the algorithm (in mana)
Cost assigned by Blizzard (in mana)
0
1
2
3
4
5
6
7
8
9
10


undervalued cards
Cost estimated by the algorithm (in mana)
Cost assigned by Blizzard (in mana)
0
1
2
3
4
5
6
7
8
9
10


undervalued cards
overpriced cards
Cost estimated by the algorithm (in mana)
Cost assigned by Blizzard (in mana)
0
1
2
3
4
5
6
7
8
9
10

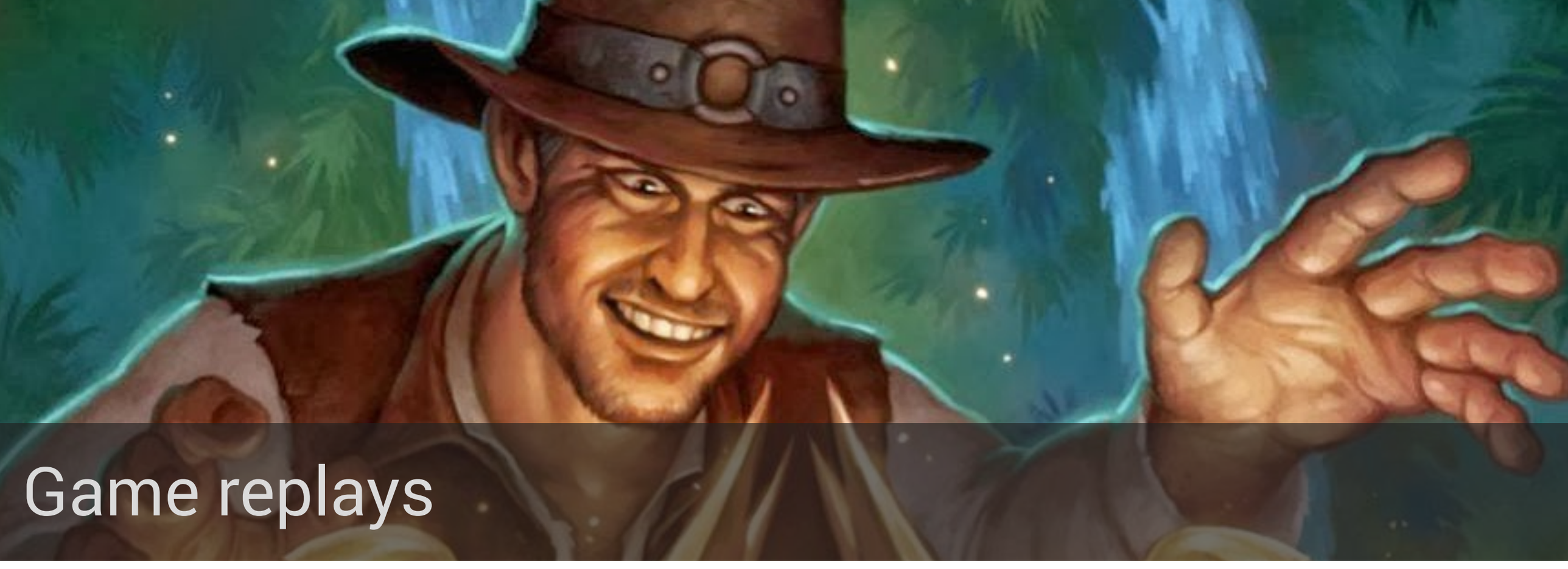


Model found a couple of clearly undervalued cards :)

Most under-valued cards (~130 cards)

Full data https://www.elie.net/tools/hearthstone/cards_value

Taking it to the next level



Game replays

- 100 000 games from May to June
- Thanks to … for it :)
- Need a longer term solution

Pricing cards with unique effects

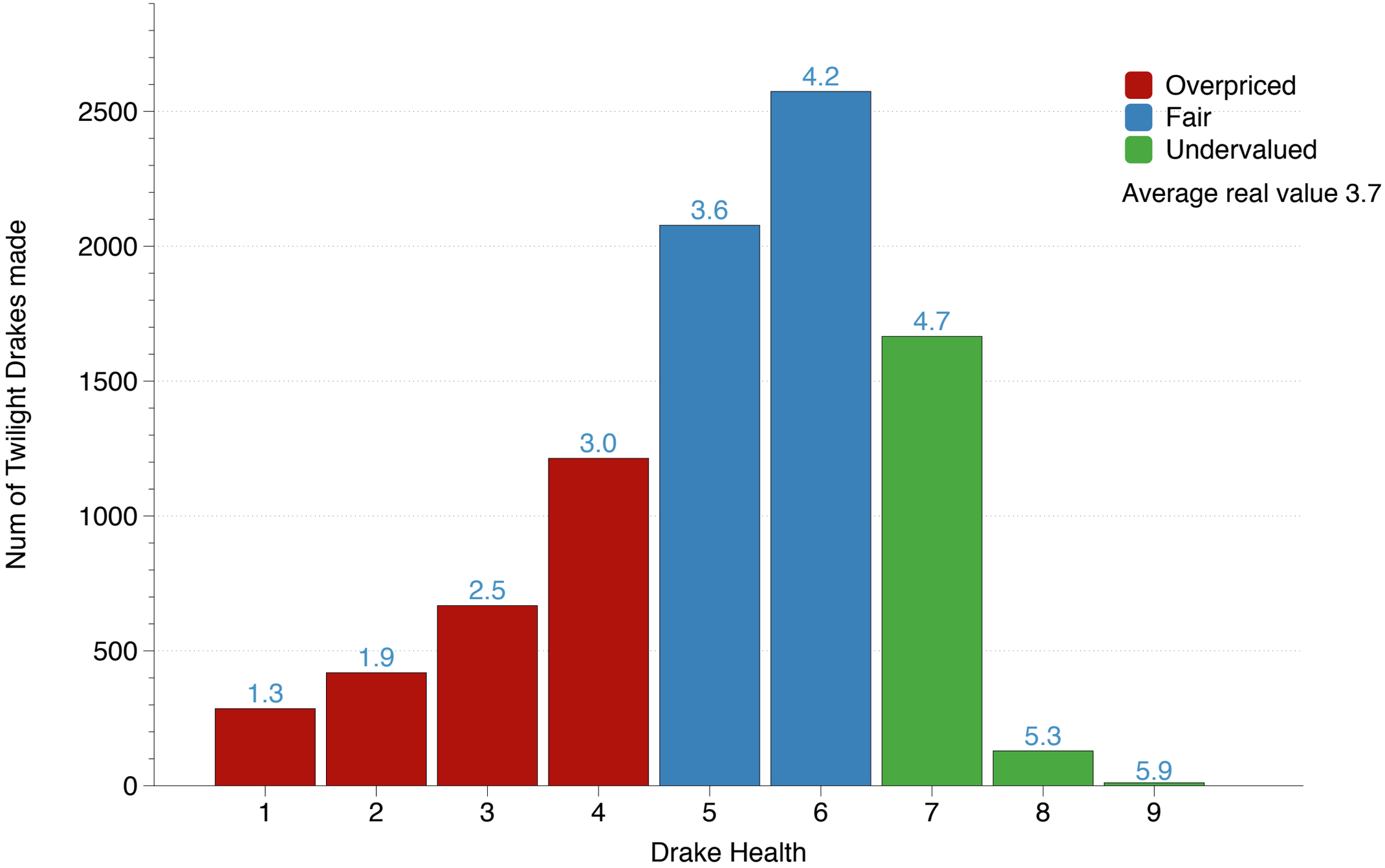
4
Twilight Drake
Battlecry: Gain +1 Health for each card in your hand.
4
Dragon
1

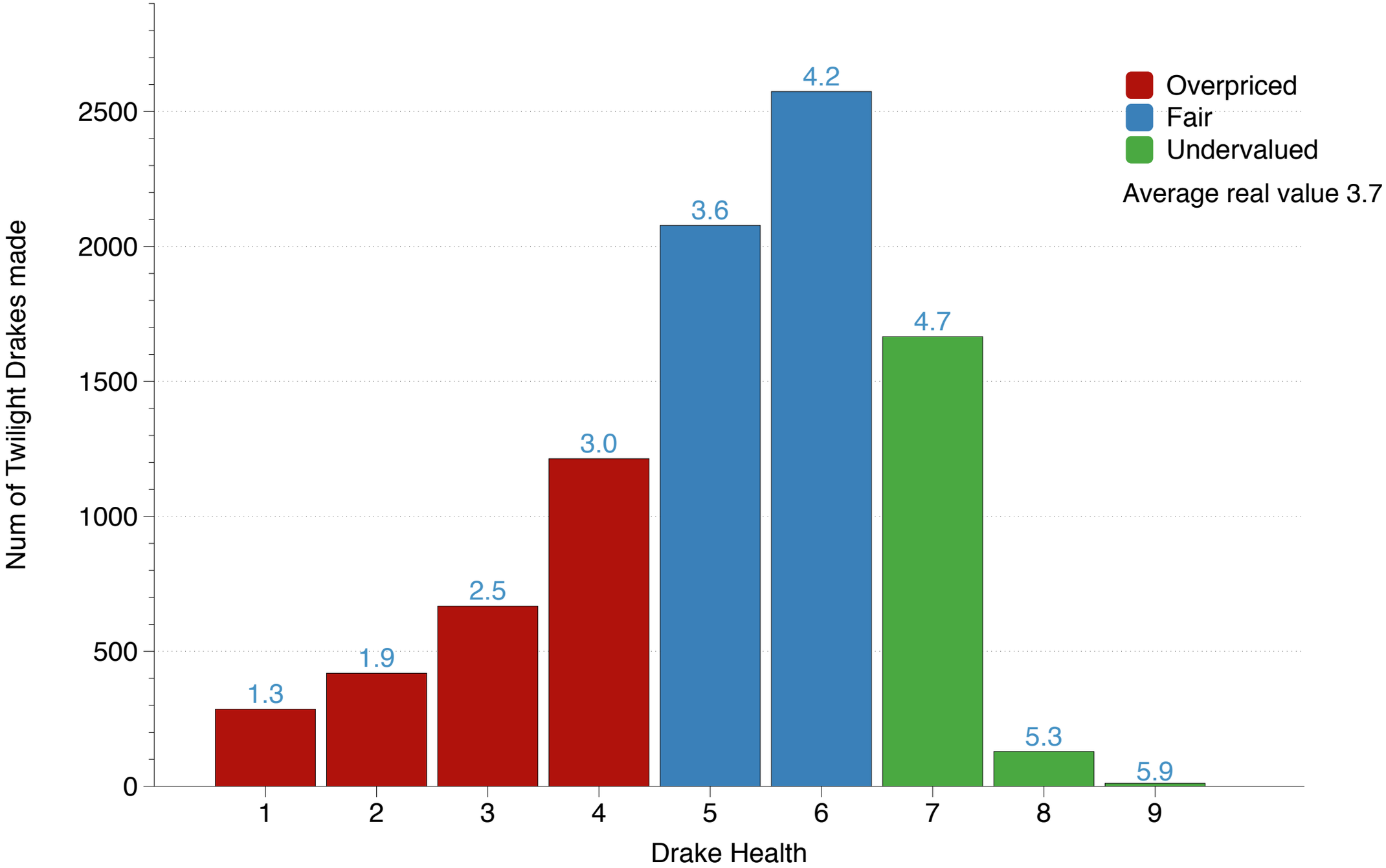
4
Twilight Drake
Battlecry: Gain +1 Health for each card in your hand.
4
Dragon
1

Cards in hand

Cards in hand →

Health	Real Value
1	1.3
2	1.9
3	2.5
4	3
5	3.6
6	4.1
7	4.7
8	5.3
9	5.9


Num of Twilight Drakes made
2500
2000
1500
1000
500
0
1.3
1.9
2.5
3.0
3.6
4.2
4.7
5.3
5.9
1
2
3
4
5
6
7
8
9
Drake Health
Overpriced
Fair
Undervalued
Average real value 3.7



Twilight drake price is fair

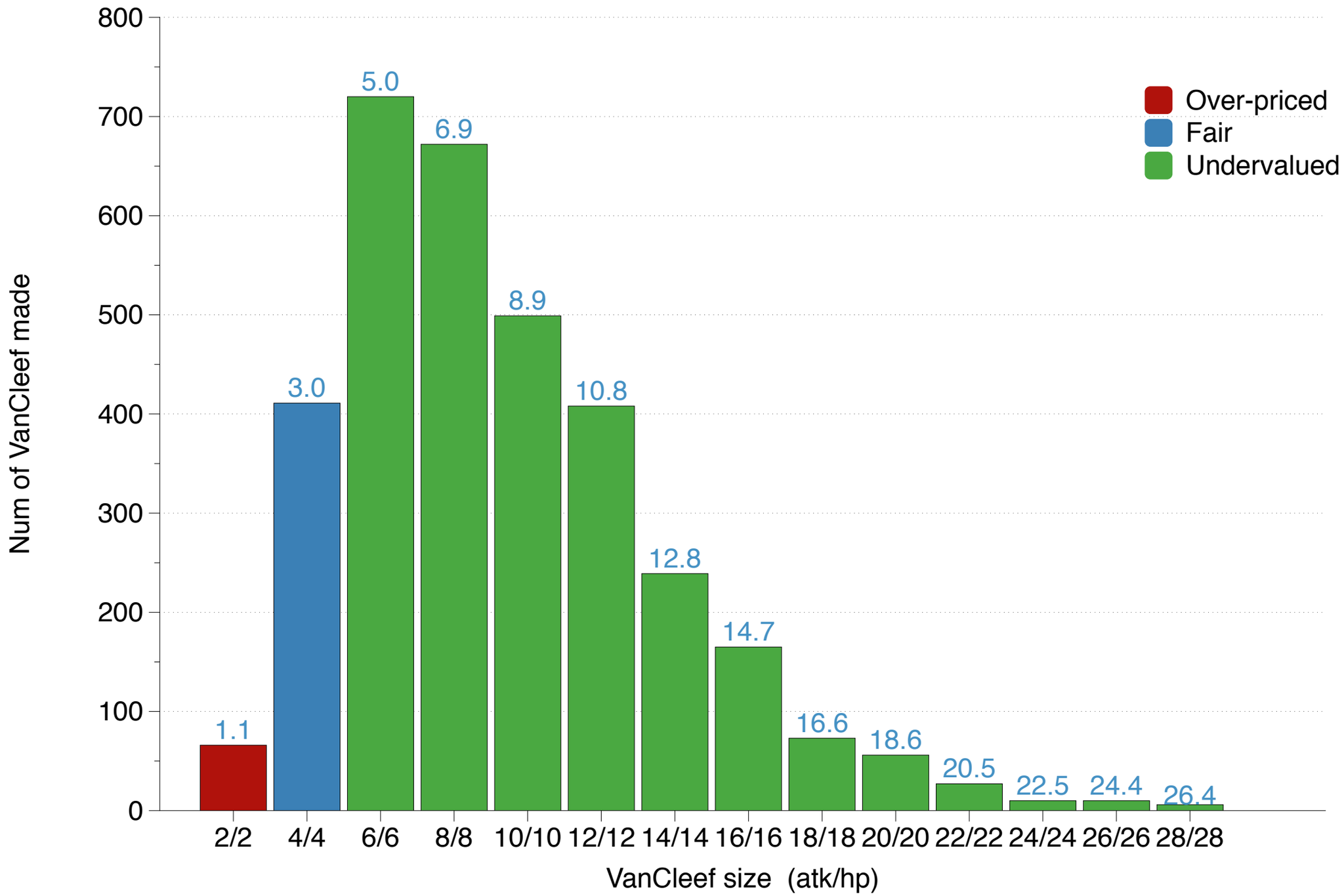
3
Edwin VanCleef
Combo: Gain +2/+2 for each card played earlier this turn.
2
2

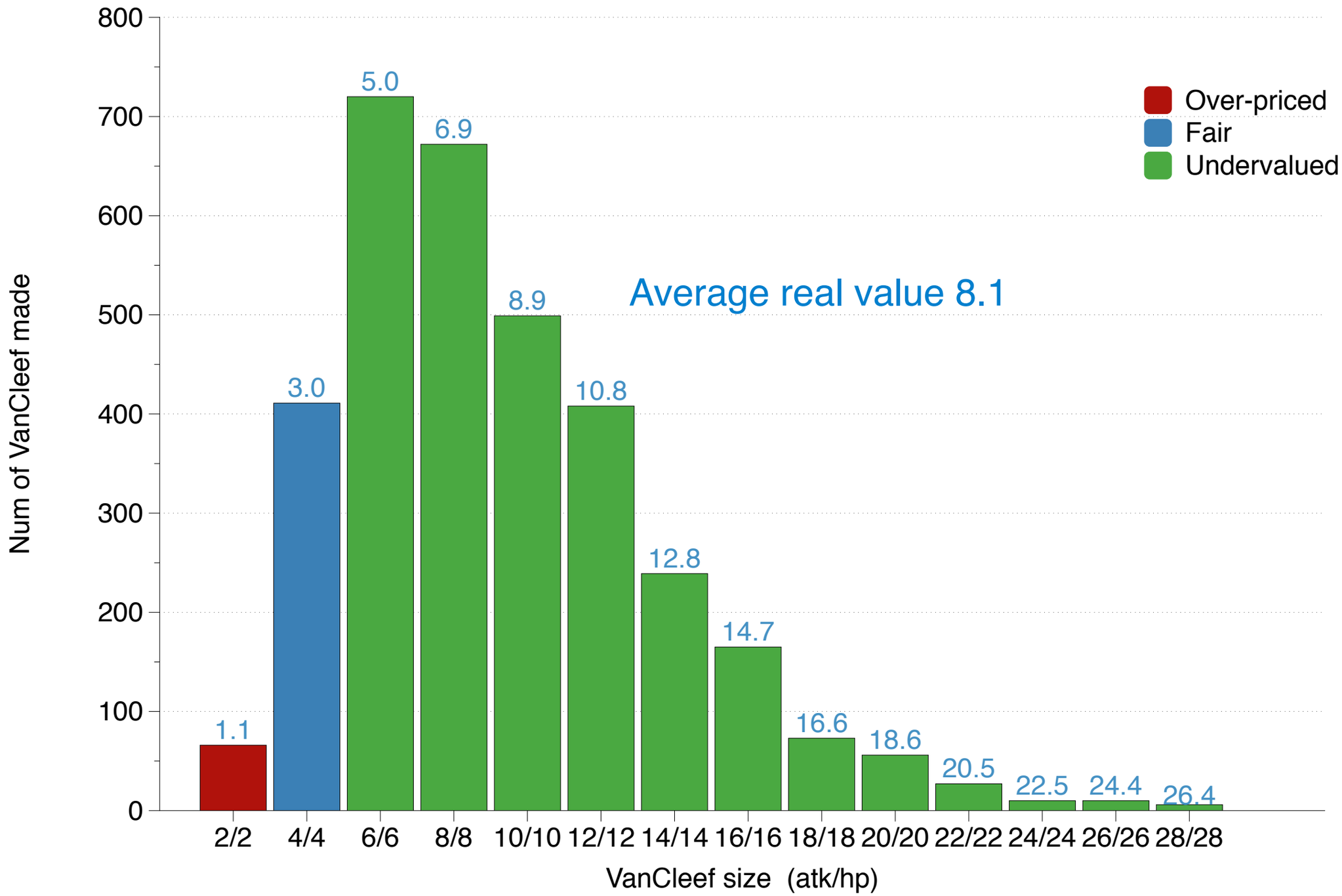
3
Edwin VanCleef
Combo: Gain +2/+2 for each card played earlier this turn.
2
2

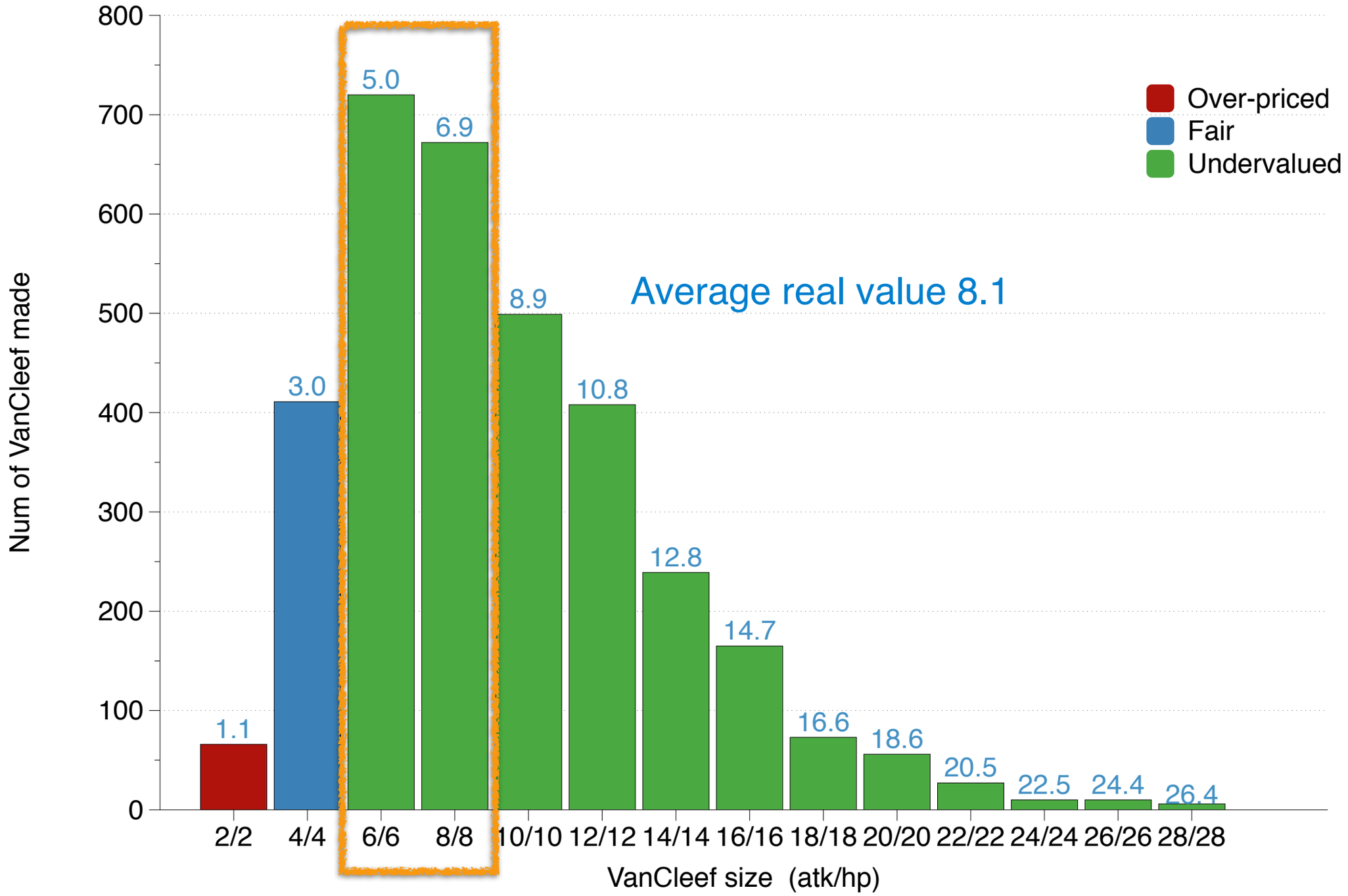
Cards played this turn

Cards played this turn

Edwin size	Real value
2/2	1.09
4/4	3.04
6/6	4.98
8/8	6.92
10/10	8.87
12/12	10.51
14/14	12.75
16/16	14.70
18/18	16.64
20/20	18.58
22/22	20.53


Num of VanCleef made
800
700
600
500
400
300
200
100
0
Over-priced
Fair
Undervalued
1.1
3.0
5.0
6.9
8.9
10.8
12.8
14.7
16.6
18.6
20.5
22.5
24.4
26.4
2/2
4/4
6/6
8/8
10/10
12/12
14/14
16/16
18/18
20/20
22/22
24/24
26/26
28/28
VanCleef size (atk/hp)


Num of VanCleef made
800
700
600
500
400
300
200
100
0
Over-priced
Fair
Undervalued
Average real value 8.1
1.1
3.0
5.0
6.9
8.9
10.8
12.8
14.7
16.6
18.6
20.5
22.5
24.4
26.4
2/2
4/4
6/6
8/8
10/10
12/12
14/14
16/16
18/18
20/20
22/22
24/24
26/26
28/28
VanCleef size (atk/hp)



VanCleef is undervalued, a fair price is probably between 5 and 7 mana

7
Flamestrike
Deal 4 damage to all enemy minions.

7
Flamestrike
Deal 4 damage to all enemy minions.

Num Minions

Num Minions

Damage	Value
4	6.5
8	13.9
12	21.3
16	28.6
20	36.0
24	43.4
28	50.7

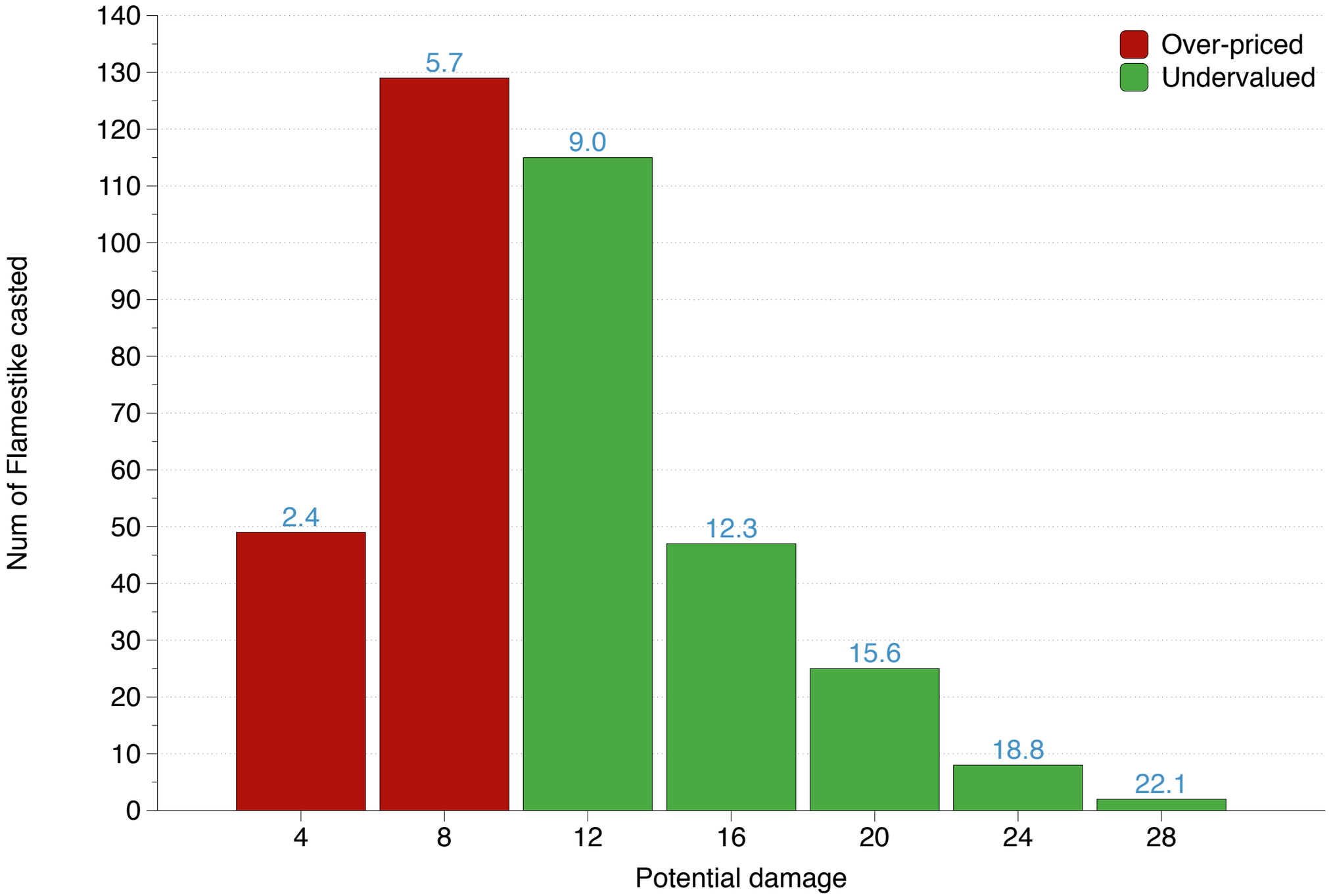
Board damage coeff

Num Minions

Num Minions

Damage	Real Value
4	2.43835
8	5.71835
12	8.99835
16	12.27835
20	15.55835
24	18.83835
28	22.11835

Normal damage coeff



Flamestrike price is fair

Don’t split board damage and single damage coeff

Predicting opponent deck

Dashboard
Started 03:13PM
MANA ADVANTAGE
3
DRAW ADVANTAGE
1
HAND ADVANTAGE
3
Me - ZOO NAXX
T P D
0 Soulfire 2 0 0
1 Abusive Sergeant 2 0 0
1 Argent Squire 2 1 0
1 Elven Archer 1 0 0
1 Flame Imp 2 1 0
1 Power Overwhelming 1 0 0
1 Voidwalker 2 1 0
2 Dire Wolf Alpha 2 0 0
2 Haunted Creeper 2 0 0
2 Knife Juggler 2 0 0
2 Nerubian Egg 2 0 0
3 Blood Knight 1 0 0
3 Harvest Golem 2 0 0
3 Void Terror 1 0 0
4 Dark Iron Dwarf 1 0 0
4 Defender of Argus 2 0 0
4 Leeroy Jenkins 1 0 0
5 Doomguard 2 0 0
My opponent -
T P D
0 Backstab 1 0 1
Predicted deck
0 Backstab 66.67
1 Deadly Poison 20.86
2 Eviscerate 4.19
2 Betrayal 1.14
1 Sinister Strike 1.05
6 Holy Fire 0.67
Elie Bursztein
3/2
29
2
0
Backstab
Deal 2 damage to an undamaged minion.
ENEMY TURN
Cealtea
27
0/2
Dire Wolf Alpha
Soulfire
Deal 4 damage.
1
3:14 PM

Our tool :)

Dashboard
Started 03:13PM
MANA ADVANTAGE
3
DRAW ADVANTAGE
1
HAND ADVANTAGE
3
Me - ZOO NAXX
T P D
0 Soulfire 2 0 0
1 Abusive Sergeant 2 0 0
1 Argent Squire 2 1 0
1 Elven Archer 1 0 0
1 Flame Imp 2 1 0
1 Power Overwhelming 1 0 0
1 Voidwalker 2 1 0
2 Dire Wolf Alpha 2 0 0
2 Haunted Creeper 2 0 0
2 Knife Juggler 2 0 0
2 Nerubian Egg 2 0 0
3 Blood Knight 1 0 0
3 Harvest Golem 2 0 0
3 Void Terror 1 0 0
4 Dark Iron Dwarf 1 0 0
4 Defender of Argus 2 0 0
4 Leeroy Jenkins 1 0 0
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My opponent -
T P D
0 Backstab 1 0 1
Predicted deck
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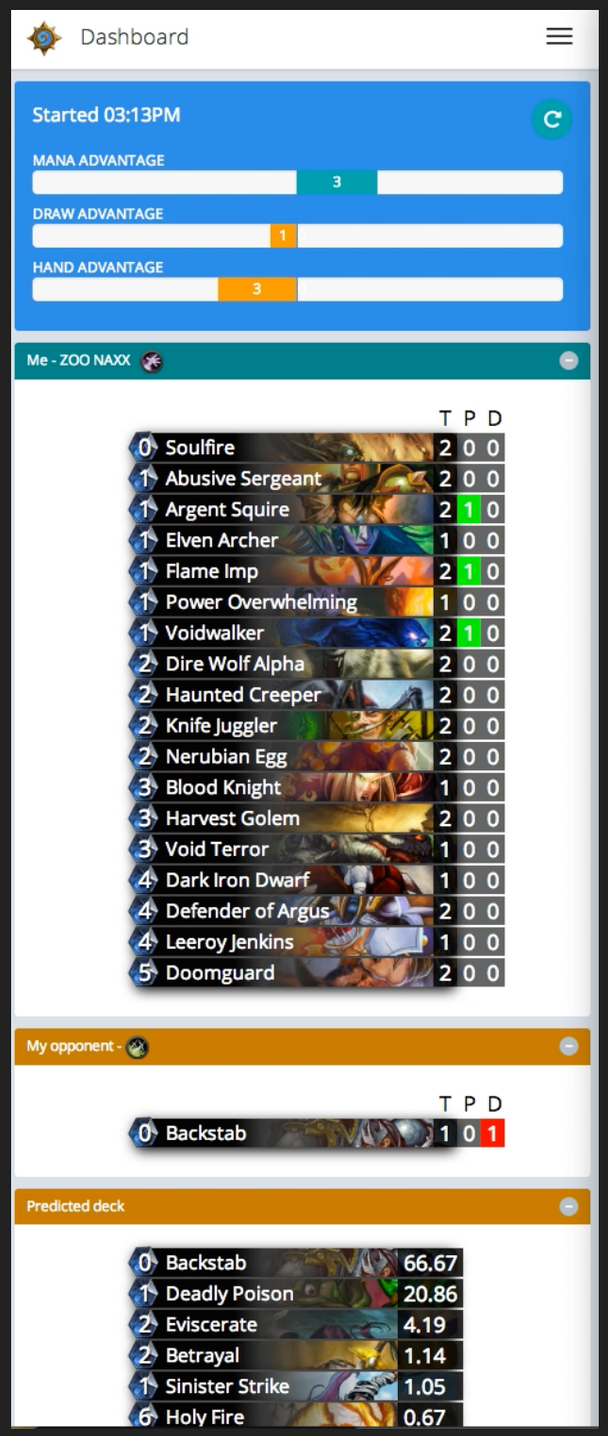
Deal 2 damage to an undamaged minion.
ENEMY TURN
Cealtea
27
0/2
1
3:14 PM
Dire Wolf Alpha
Soulfire
Deal 4 damage.

Real time dashboard

Game metrics

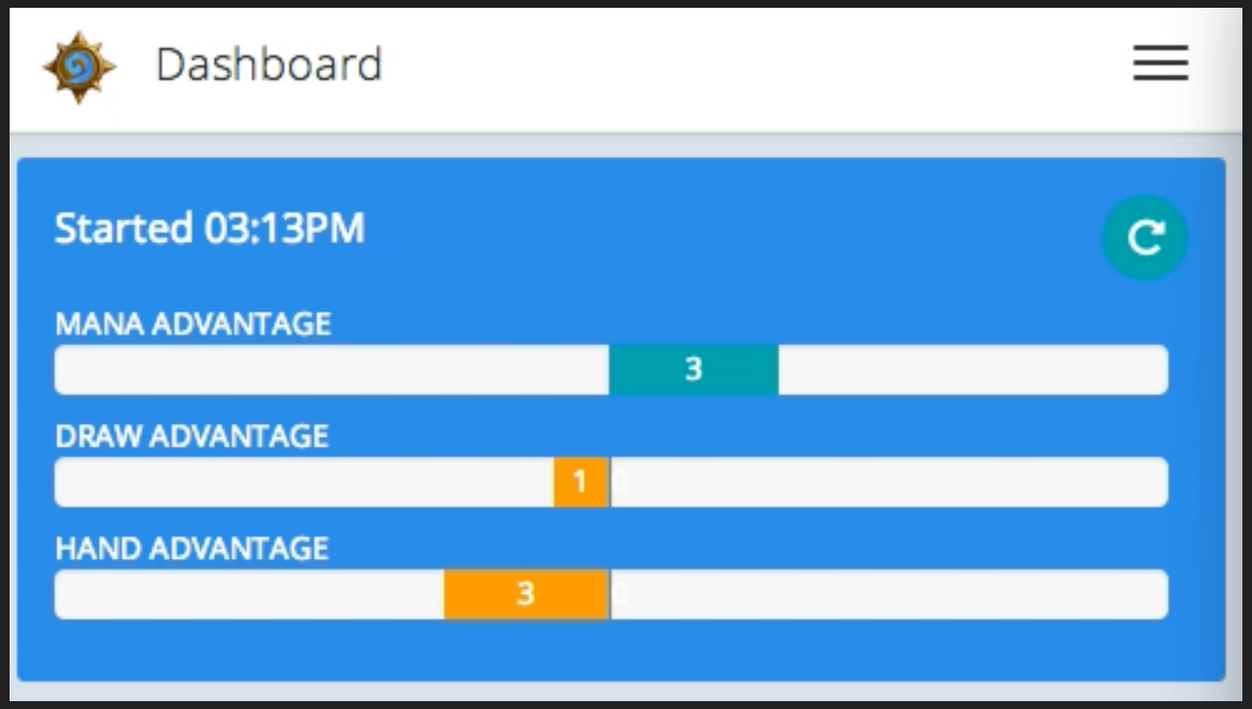


My deck with card tracking

Opponent cards played so far

Opponent next cards prediction

Game metrics



Me

Opponent

My deck

Me - ZOO NAXX

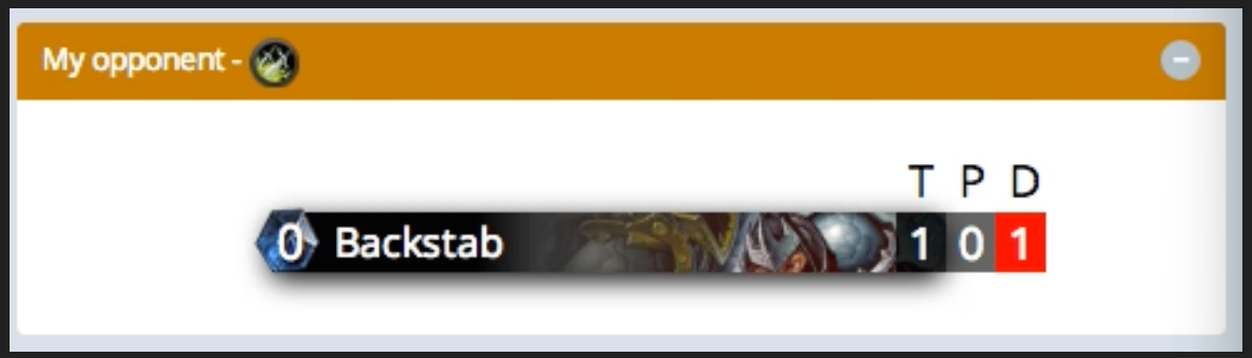
		T	P	D
0	Soulfire	2	0	0
1	Abusive Sergeant	2	0	0
1	Argent Squire	2	1	0
1	Elven Archer	1	0	0
1	Flame Imp	2	1	0
1	Power Overwhelming	1	0	0
1	Voidwalker	2	1	0
2	Dire Wolf Alpha	2	0	0
2	Haunted Creeper	2	0	0
2	Knife Juggler	2	0	0
2	Nerubian Egg	2	0	0
3	Blood Knight	1	0	0
3	Harvest Golem	2	0	0
3	Void Terror	1	0	0
4	Dark Iron Dwarf	1	0	0
4	Defender of Argus	2	0	0
4	Leeroy Jenkins	1	0	0
5	Doomguard	2	0	0

T = Total

P = Played

D = Dead

My opponent



T = Total

P = Played

D = Dead

Predictions

Game data from

Game data from

Sniff packets

Game data from

Sniff packets

OCR

Game data from

Sniff packets

OCR

Debug log

Game data from

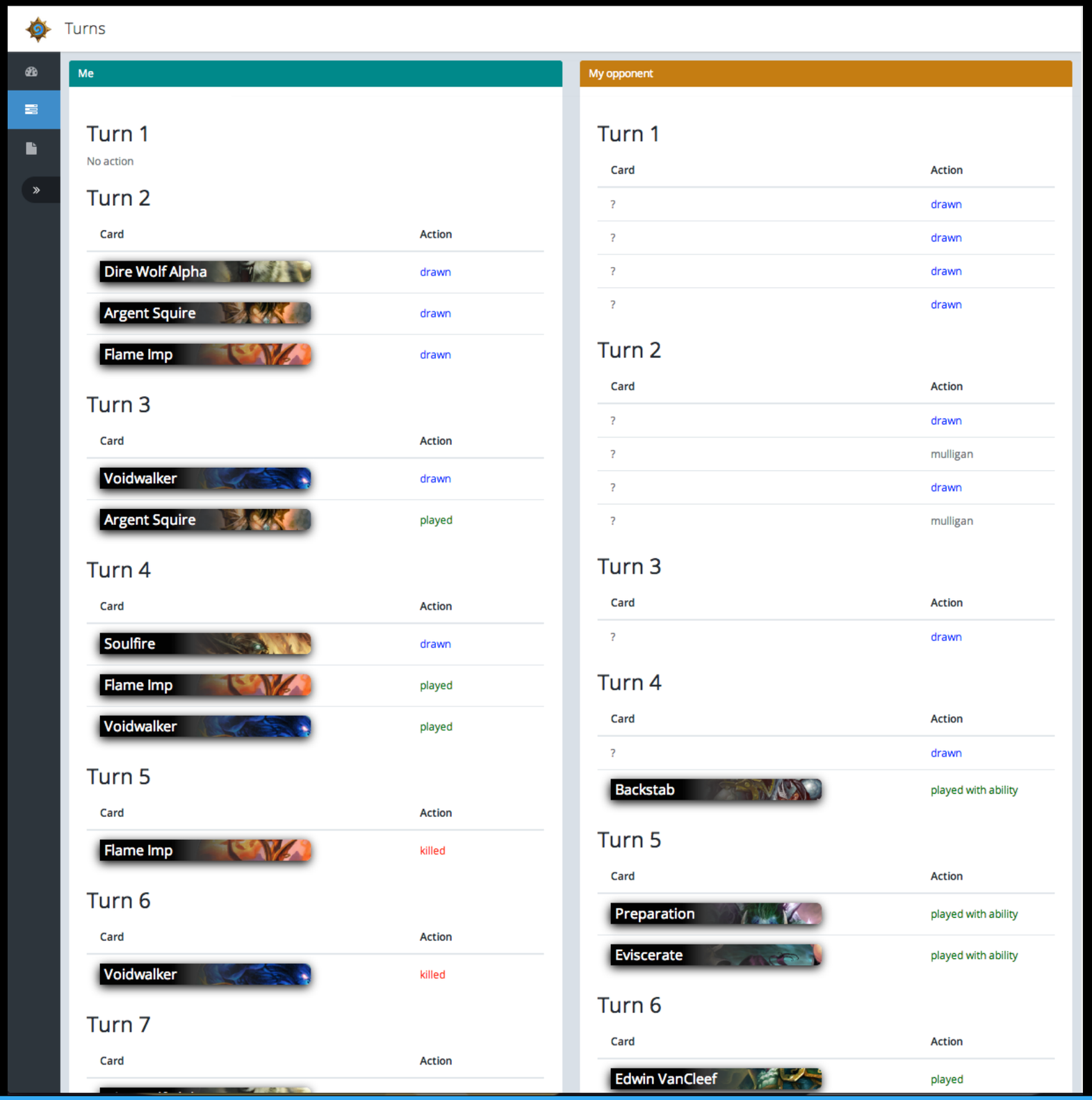
Sniff packets

OCR

Debug log

Real logs from Blizzard like in WoW ?

Turn by Turn History



Available on Github

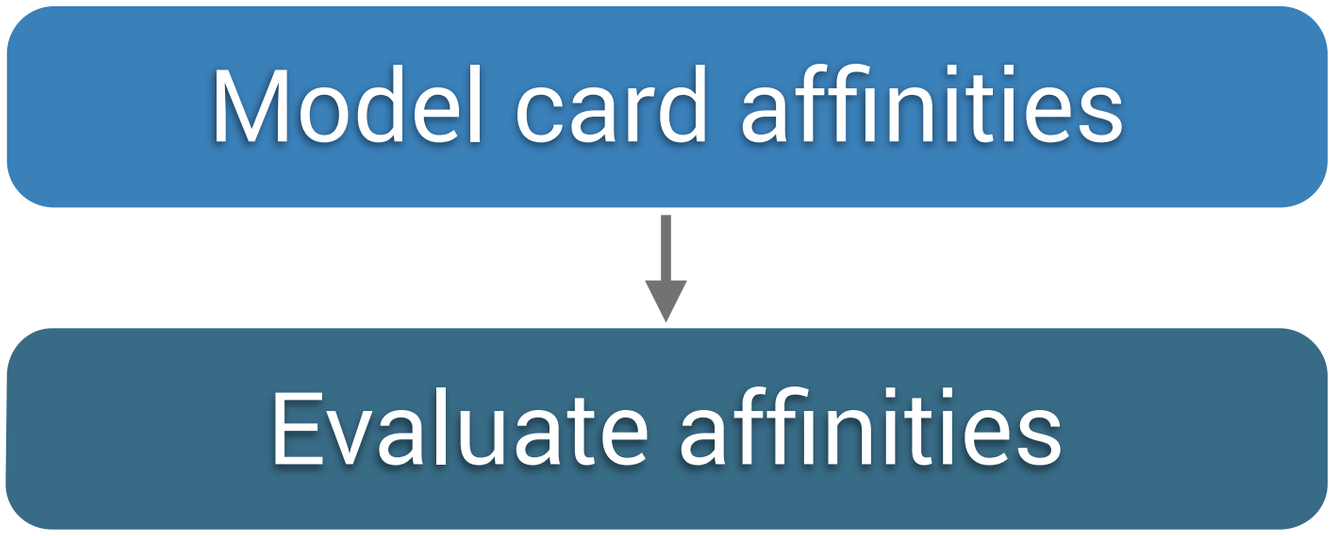
LightWind/hearthstone-dashboard

Naxx new cards made the meta to unstable to be predicted accurately for now

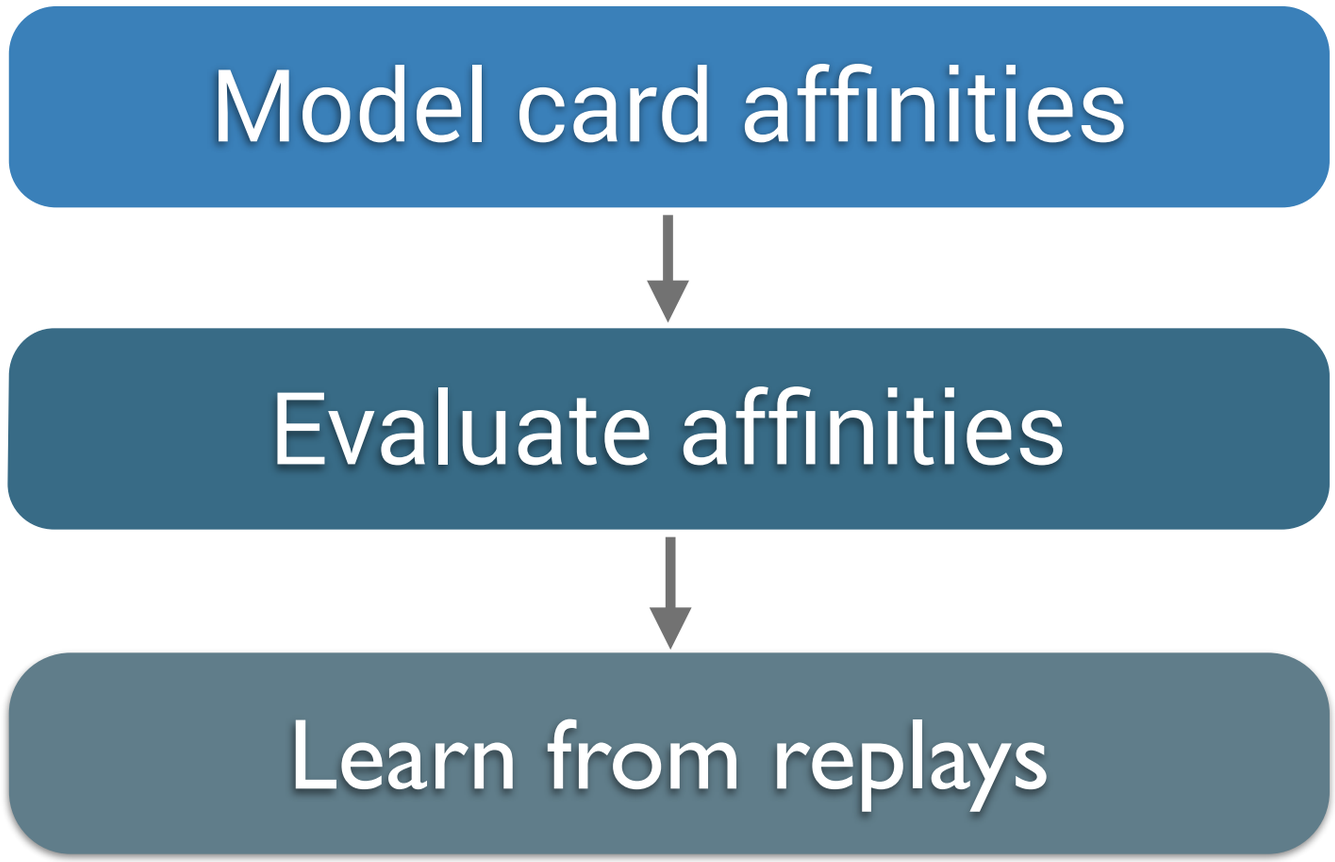
Anatomy of our prediction system

Model card affinities

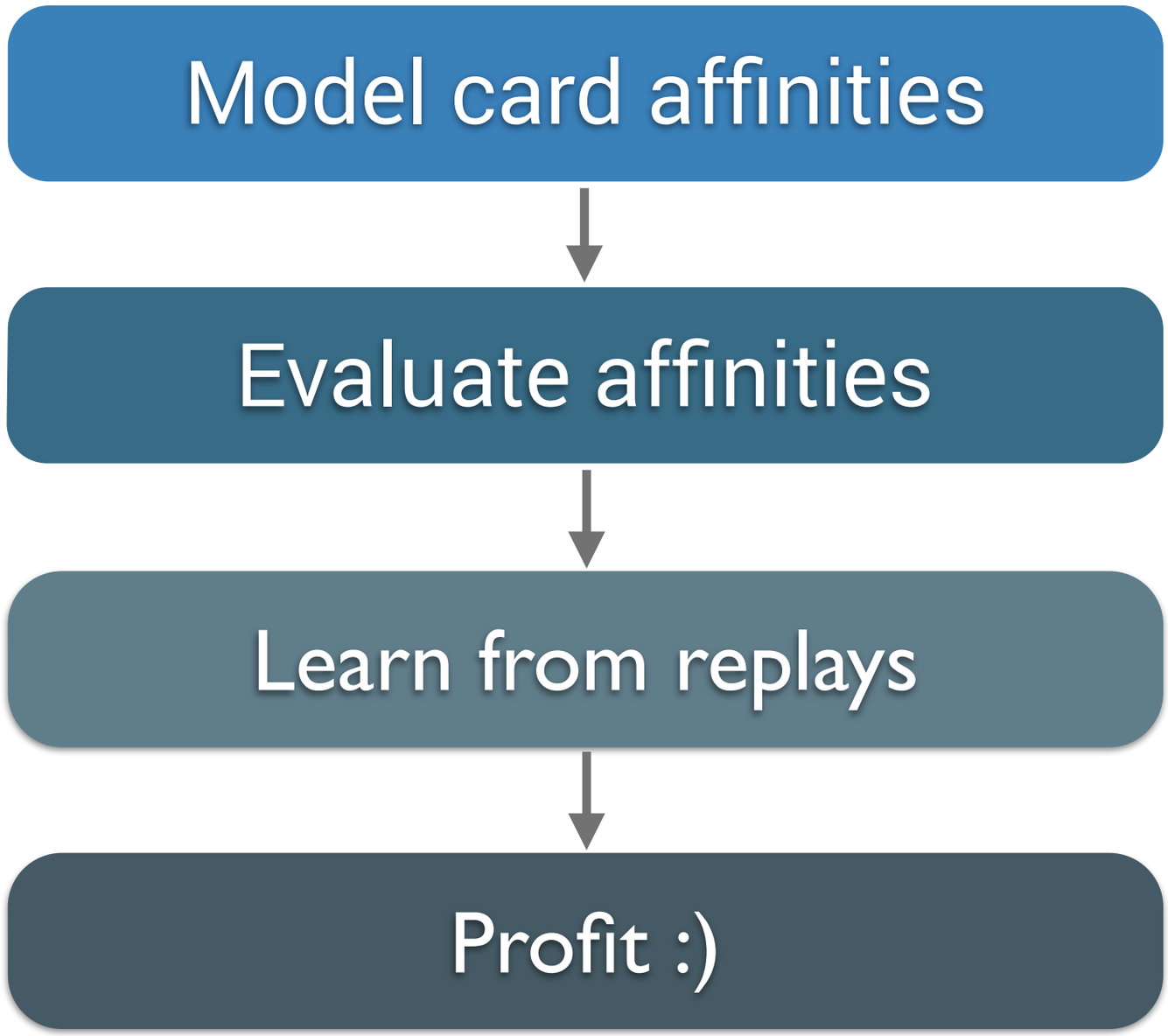
Anatomy of our prediction system



Anatomy of our prediction system



Anatomy of our prediction system



Modeling cards affinities

Cards bigrams

Cards bigrams

Cards bigrams

Cards un-ordered bigrams

Evaluate cards affinities

Played

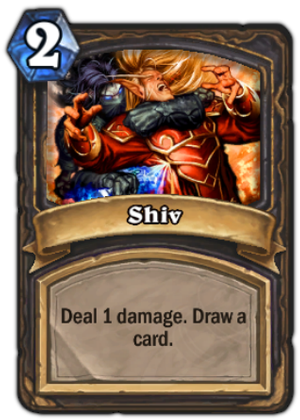
Played

1
Deadly Poison
Give your weapon +2 Attack.

2
Shiv
Deal 1 damage. Draw a card.

Played

Bi-grams



Played

Bi-grams

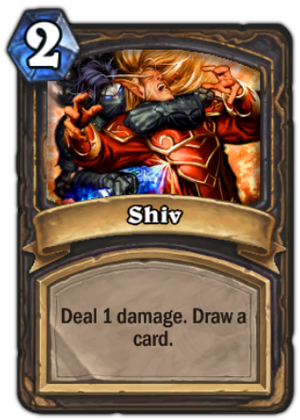
500

Played

Bi-grams

500

350



Played

Bi-grams

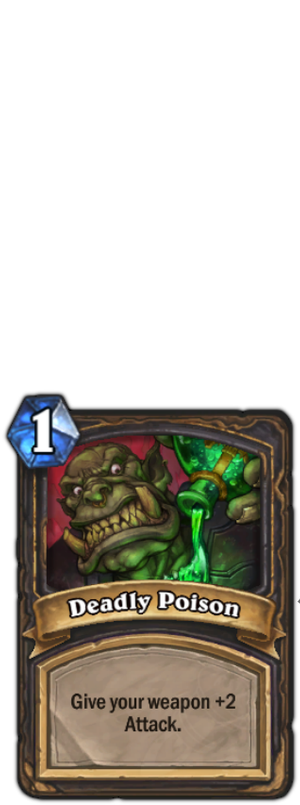
500

350

400

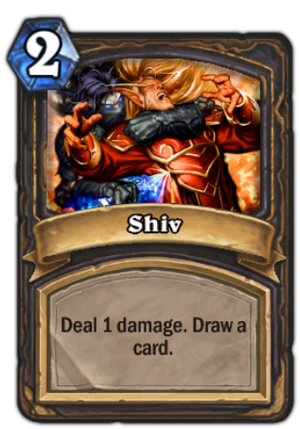
Played

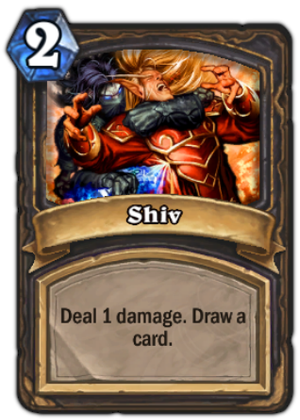
Bi-grams



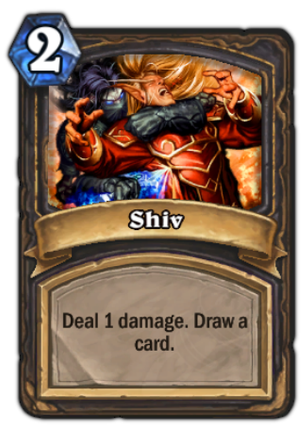
500

350





400



400

Played

Bi-grams

Ranked Predictions

500

350

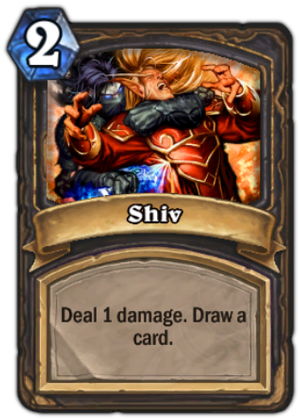
400

400

Played

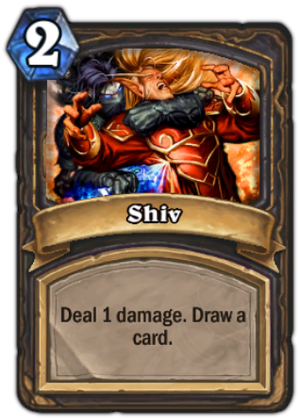
Bi-grams

Ranked Predictions

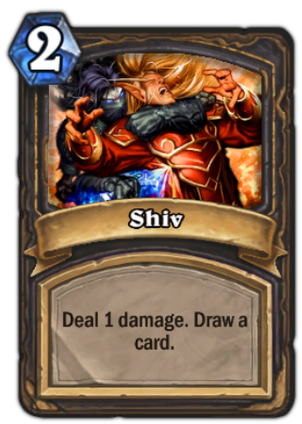


500

350



400



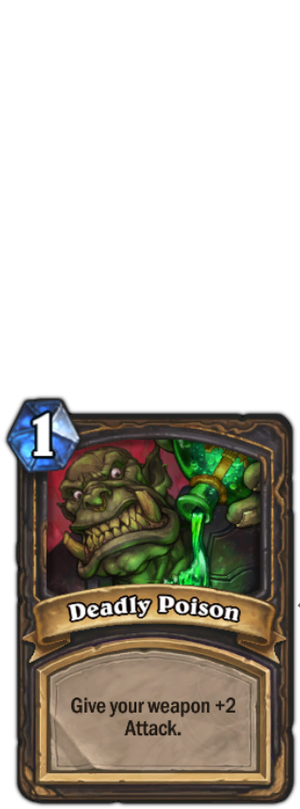
400

750

Played

Bi-grams

Ranked Predictions



500

350

400

400

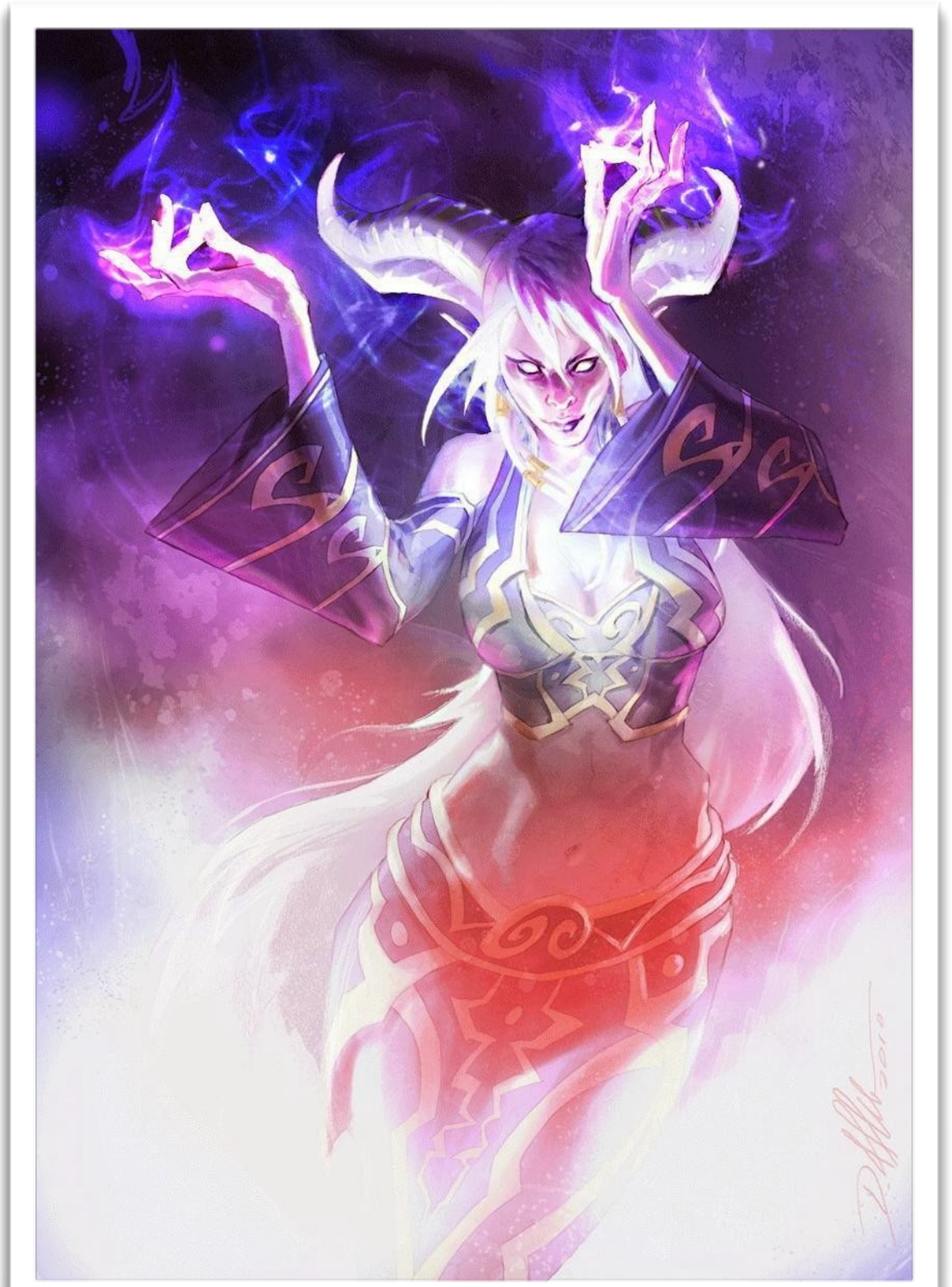
750

500

Played

Bi-grams

Ranked Predictions



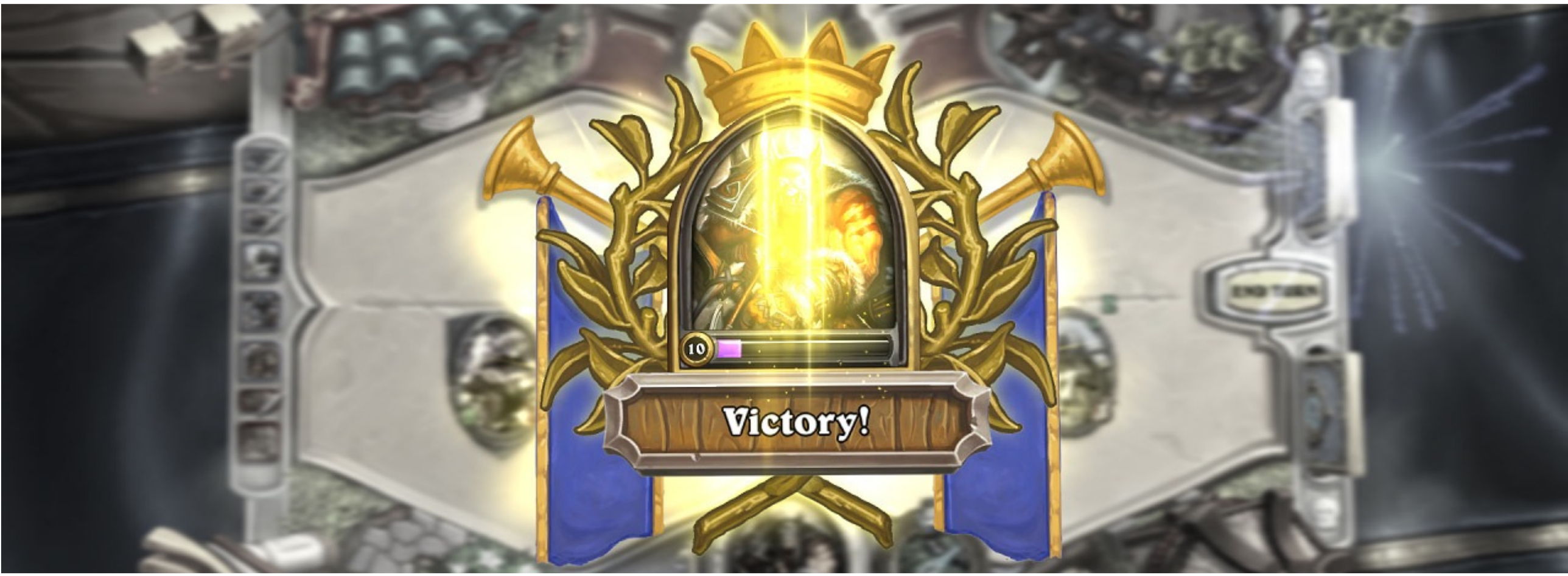
Training and evaluation

Training: 45000 replays

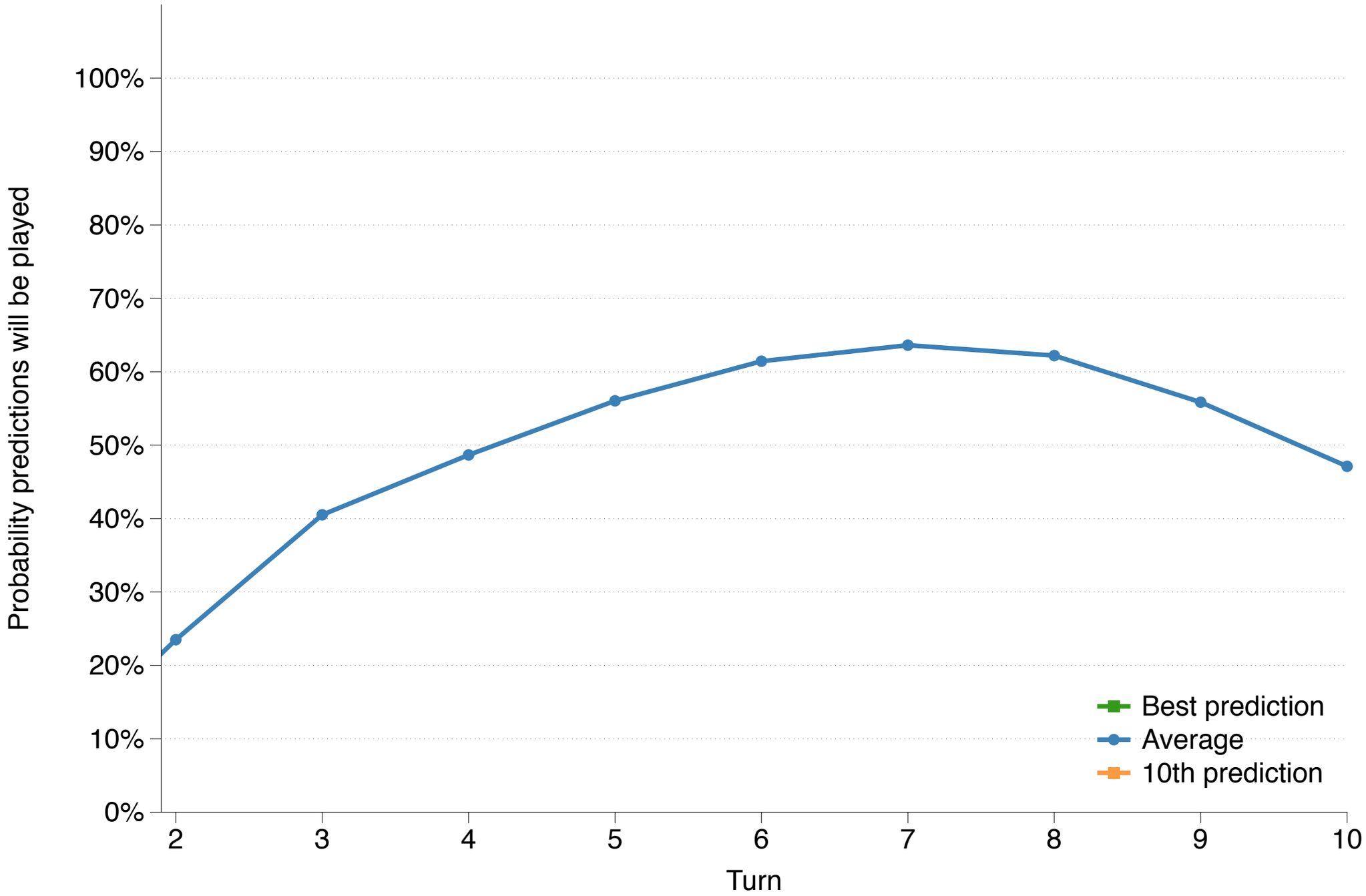
Testing : 5000 replays

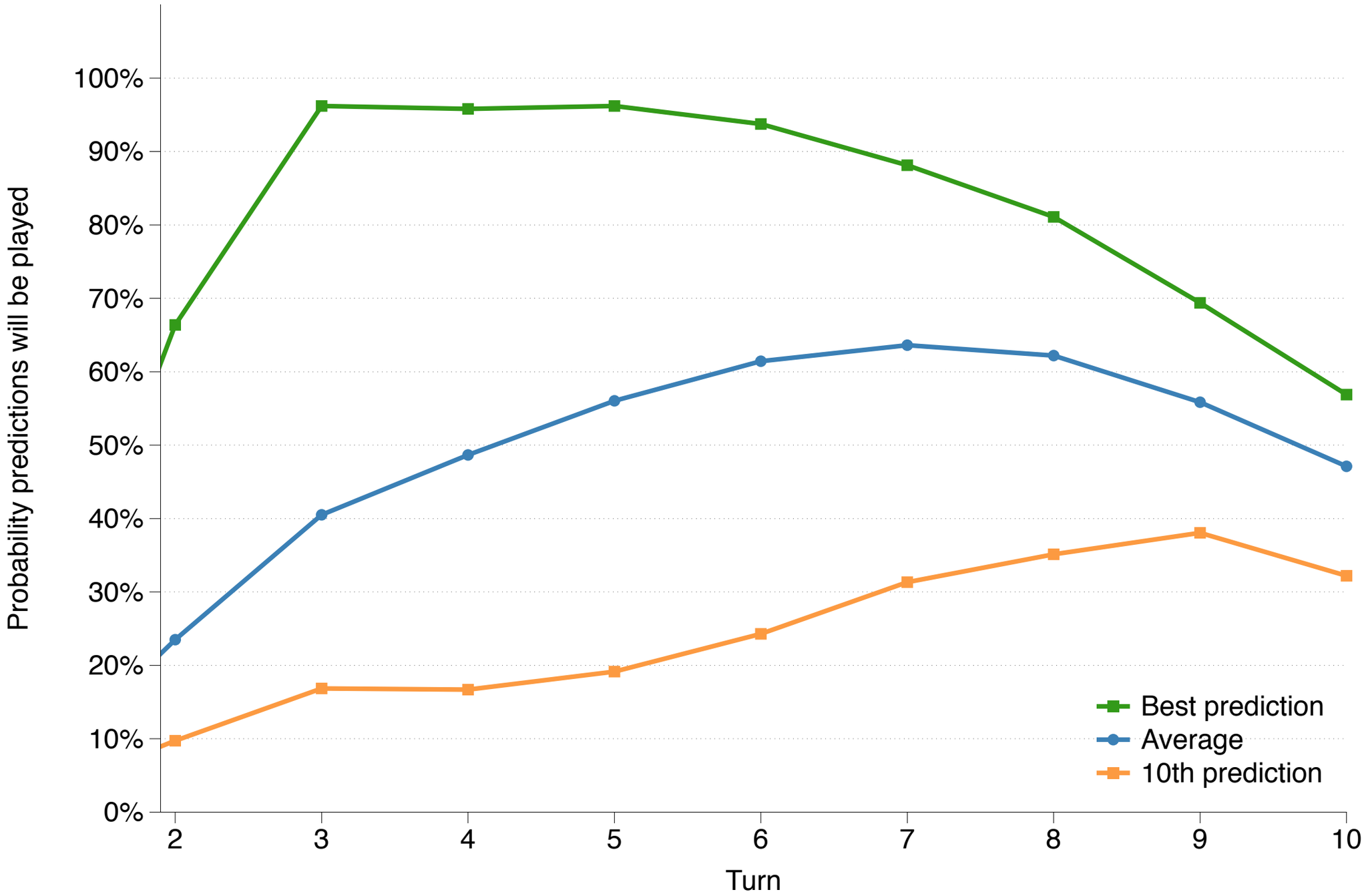
1 model per class

Suspense is killing me



97% success rate for best prediction by turn 3


Probability predictions will be played
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
2
3
4
5
6
7
8
9
10
Turn
Best prediction
Average
10th prediction


Probability predictions will be played
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
2
3
4
5
6
7
8
9
10
Turn
Best prediction
Average
10th prediction

What's next?

Predicting game outcome

How to optimize deck for mana-throughput

Hero powers comparison

Comparing decks types

Thank you!

https://www.elie.net/hs & @elie/@cealtea on Twitter